Monetary policy, financial stability and inequality [1]

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Abstract

This paper examines the joint design of monetary policy and capital controls in an environment with a motive for both financial stability and price stability. I build an equilibrium business cycle model with a current-price collateral constraint, household heterogeneity due to a limited financial market participation, and nominal rigidity. I show that, in the absence of credit frictions (i.e., the collateral is never binding), the monetary authority under the discretionary monetary policy has an incentive to deviate implementing price stability (the divine coincidence does not hold). In addition, I show that in the case of financial instability due to credit frictions, the monetary authority under the discretionary monetary policy should adopt a prudential monetary policy only if capitals flows are free. This prudential monetary policy is exacerbated by household inequality. In the absence of a working capital loan procyclical monetary policy is never optimal.

Keywords: Financial stability , inequality, monetary policy, capital controls.

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1 Introduction

In the aftermath of the global financial crisis, capital controls have been recommended as a macroprudential tool to alleviate the severity and frequency of financial instability. Additionally, many advanced economies and emerging markets have adopted an inflation targeting[3] framework that aims to achieve price stability. This paper studies the joint design of monetary policy and capital controls in an environment with inequality and motives for both financial and price stability. Specifically, the paper examines the optimal policy response of the monetary authority to financial instability arising from credit frictions, and considers the role of capital flows and household inequality in shaping the optimal policy. I show that in cases of financial instability due to credit frictions, the monetary authority under discretionary monetary policy should only adopt a prudential monetary policy if capital flows are free, and that inequality exacerbates the need for such a policy.

To study the joint design of monetary policy and capital control, I extend one of my previous papers[4] to incorporate price rigidity. I enrich a standard dynamic stochastic general equilibrium model that features an occasionally binding collateral constraint with limited household heterogeneity. The model features two types of households. The first type comprises households who participate in the financial market and have access to the capital and bond market. These households are called *"asset holders"*. The second type of household comprises those who do not participate in either the capital or bond market. These households, called *"hand-to-mouth"* consumers, consume all of their labor income plus any additional transfers. The small open economy faces shocks to its productivity, the real interest rate, and the price of imported inputs. I also introduce a financial shock. A financial shock — a drop to the loan-to-value ratio — consists of a drop during a financial crisis in the fraction of the total value of physical assets that households can pledge as collateral. The model economy nests the model in Bianchi and Mendoza (2018).

I characterize the optimal monetary policy under discretion in the absence of credit frictions.

[3] In an inflation targeting framework, the central bank forecasts the future path of inflation and compares it with the target inflation rate. The difference between the forecast and the target determines the degree to which monetary policy needs to be adjusted. Therefore, the central bank's policy response depends on how far the forecasted inflation rate is from the target rate.

[4] The paper is titled Sudden stops, asset prices: the role of financial market participation and can be downloaded here

The result suggests that there is a trade-off between price stability and output stabilization. The monetary authority under the discretionary monetary policy has an incentive to deviate from implementing price stability (i.e., the divine coincidence does not hold). A central bank deviates from its price stability objective because of a concern of inequality. My result is consistent with Acharya et al. (2020) who find in a Heterogeneous Agents New Keynesian (HANK) framework, that a concern for inequality leads the monetary-maker to weigh an economic activity stabilization more than a price stabilization.

I also characterize the optimal monetary policy under discretion and with free capital flow (i.e., no capital control). Whether the central banks should conduct a contractionary monetary policy or an expansionary monetary policy during the financial crisis is ambiguous. However, in the absence of a working capital loan, the central banks should conduct an expansionary monetary policy during the financial crisis. By lowering domestic nominal interest rates during the crisis, investors demand lower premium on their domestic physical asset which raises the asset price and relaxes the collateral constraint. In normal time (i.e., when the collateral does not bind), If the monetary authority anticipates financial crises in the future, they are more likely to conduct an expansionary monetary policy. By lowering domestic nominal interest, it lowers the demand for foreign bond and reduce vulnerability to capital inflows in the future. Coulibaly (2018) finds similar result in two consumption goods model, tradable and non-tradable goods. He shows that a sufficient condition to conduct an expansionary monetary policy in normal time is when the intra-temporel elasticity of substitution is greater than the inter-temporel elasticity substitution.

The presence of household heterogeneity distorts the discretionary monetary policy without capital controls in three dimensions. First, in the absence of credit friction, the central banks have an incentive to deviate from the price stability for inequality concern. Second, inequality amplifies the ex-ante financial motive response for monetary policy. The monetary policy should be more expansionary in normal time to mitigate the distributional impacts of the financial crisis. Third, inequality may affect qualitatively the monetary policy during the financial crisis. The monetary policy is less likely to be contractionary during the financial crisis.

My paper mainly relates to the literature that studies the aggregate effects of a sudden stop (see, for example, Arellano and Mendoza (2002), Chari et al. (2005), Mendoza (2006), Calvo et al. (2006). Mendoza (2010), and Korinek and Mendoza (2014)). My paper is closely related to Mendoza (2010), who studies how an endogenous binding collateral can trigger the economy within standard business cycle moments. I have three contributions to this literature. First, I introduce limited financial market participation where a fixed share of households do not participate in the financial market. This characterization of the economy is closer to that of emerging markets and helps us to explain the observed gap in the decline in the asset price during sudden stops between emerging markets and advanced economies. Second, my work studies the optimal time-consistent solution and rationalizes the prevalent use of capital control in emerging markets. Finally, I introduce price rigidity to study the optimal monetary policy under discretion.

My work is also related to recent literature that studies the optimal policy in a financial crisis model. These papers include Caballero and Krishnamurthy (2004), Bianchi (2011), Bengui (2014), Bengui and Bianchi (2018), Bianchi and Mendoza (2018), and Arce et al. (2019). I contribute to this literature by taking into account household heterogeneity in the financial market and show that it is possible to address financial instability without raising inequality. My paper also relates to the literature on financial crises and macroprudential policy. This literature has shown how capital controls can correct pecuniary externalities that generate excessive systemic risk (e.g., Lorenzoni (2008); Bianchi (2011); Dávila and Korinek (2018)). I contribute to this literature by showing monetary policy can serve as a macroprudential policy tool.

My work is also related to the literature that studies the optimal monetary policy when the economy is prone to sudden stops (e.g., Coulibaly (2018); Devereux et al. (2019); Devereux et al. (2015); Bianchi and Coulibaly (2021) ; Davis and Presno (2017); Chang et al. (2015)). I contribute to this literature by studying how inequality exacerbates the challenges of implementing prudential monetary policy. Previous research, such as Coulibaly (2018) in a two consumption goods model, has shown that procyclical monetary policy is optimal when both goods are complements. However, in my model where consumers have access to only one consumption good and no working capital loans, procyclical monetary policy is never optimal.

The rest of the paper is organized as follows. Section 2 presents the model. Sections 3 presents my theoretical findings and Section 4 concludes.

2 Model with collateral constraint and household heterogeneity

I build a small open economy model with household heterogeneity and a collateral constraint. The sudden stop crisis is driven by an occasionally binding collateral constraint. There are two types of households. The first type comprises asset holder consumers who have access to the financial market through their holding of both physical assets and foreign bonds. The second type are “hand-to-mouth" consumers who do not hold any assets — neither physical assets nor foreign bonds. They consume all of their labor income plus any additional transfers from the government. Asset holders act as an entrepreneur who produces an intermediate good. The intermediate good is sold to retailers, which differentiate the good at no cost and sell to the final-good producer. I assume that each retailer set on a monopolistically competitive market, the price of its own differentiated good subject to a convex adjustment cost a la Rotemberg (1982) . The retailers' profits are redistributed to asset holders. The final good producer set the price of the aggregate good on a perfectly competitive market.

2.1 Entrepreneur and asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure 1. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}), \tag{1}$$

where $\mathbb{E}_0$ is the expectations operator; β is the discount factor; C_{1t} is consumption; $u(.)$ is the utility function which is a standard concave, twice continuously differentiable function that satisfies the Inada condition.

Households produce final goods using three inputs, which are physical assets k_t, intermediate goods v_t, and labor demnand L_t. The production technology is such that $y = A_t F(k_t, L_t, v_t)$, where F is a twice continuously differentiable, concave production function and $A_t = A\exp(\epsilon_t^A)$ is

TFP subject to a random shock ϵ_t^A. This shock follows a stationary Markov process. Intermediate goods are traded in competitive world markets at a price p_t^v. The price $p_t^v = p\exp(\epsilon_t^v)$ is subject to a random shock ϵ_t^v that follows a stationary Markov process. *Asset holder* households borrow on the foreign bond market at the real interest rate $R_t^v = R\exp(\epsilon_t^r)$, where ϵ_t^r is a random shock that follows a stationary Markov process. The budget constraint of *asset holder* households is given by

$$P_t c_{1t} + \frac{B_{t+1}}{R_t} + P_t q_t k_{t+1} = P_t^e F\left(k_t, L_t, v_t\right) - P_t p_t^v v_t - P_t w_t L_t + B_t + P_t q_t k_t - T_t. \tag{2}$$

In equation 2, q_t is the price of the physical asset k_t, R_t is the nominal interest rate, and w_t is the real wage. P_t is consumption price and P_t^e is the intermediate good price. The entrepreneur sell to retailers the intermediate good that they produce at price P_t^e. The term $P_t w_t L_{2t}$ represents the total nominal labor income paid to "hand-to-mouth" households. The term T_t is the total lump-sum taxes paid by all *asset holder* households. Lump-sum taxes are used to calibrate the average consumption inequality.

The total private debt in the economy is restrained to a fraction κ of the market value of the beginning-of-period physical asset given by

$$\frac{B_{t+1}}{R_t} - \phi P_t p_t^v v_t \geq -\kappa_t P_t q_t k_t. \tag{3}$$

On the left-hand side of (3), total private debt (in negative terms) is the sum of private debt with one-year maturity and the within-period working capital loan. On the left-hand side of (3), the term $\phi P_t p_t^v v_t$ represents the working capital loan. The working capital loan is a fraction ϕ of the total cost of intermediate inputs in advance of sales. On the right-hand side of (3), the term $\kappa_t P_t q_t k_t$ represents a fraction κ_t of the market value of the end-of-period physical asset. Only *asset holder* households who borrow in the foreign bond market face this collateral constraint. Although I do not derive the collateral constraint from an optimization problem, Bianchi and Mendoza (2018) show that this type of constraint could be obtained as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the market value of an asset owned by a defaulting debtor.

The term κ_t represents the financial shock and can be interpreted as the fraction of the total value of physical assets the households can pledge as collateral. It takes two values: a high value κ_h regime and a low value κ_l regime (time of crisis) with a switching probability between both regimes. This is consistent with the data, which suggest that the loan-to-value ratio decreases during a financial crisis. According to the loan-to-value ratio in Mexico in the 1990s, I set $\kappa_h = 0.7$ and $\kappa_l = 0.55$. The probability of staying in the low regime is set to zero to reflect the fact that the average duration of a sudden stop is one year. I then use the probability of staying in the high regime to calibrate the frequency of the financial crisis. The beginning-of-period asset k_t is used as collateral instead of the end-of-period asset k_{t+1}, and there is no labor in the working capital loan.

The *asset holder* households choose consumption, borrowing, capital, labor, and intermediate inputs to maximize their utility (1) subject to their budget constraint (2) and their borrowing constraint (3), taking prices as given. Their optimality conditions are given by

$$u'(t) = \beta R \mathbb{E}_t \left[u'(t+1) \right] + \mu_t, \tag{4}$$

$$q_t u'(t) = \beta \mathbb{E}_t \left[(d_{t+1} + q_{t+1}) \, u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1} \right], \tag{5}$$

$$X_t F_l \left(k_t, L_t, v_t \right) = w_t, \tag{6}$$

$$X_t F_v \left(k_t, L_t, v_t \right) = p_t^v + \phi \frac{\mu_t}{u'(t)} p_t^v, \tag{7}$$

where $X_t = \frac{P_t^e}{P_t}$ is the inverse of the retailer markup, $\mu_t \geq 0$ is the Lagrange multiplier on the borrowing constraint, $u'(t)$ is the partial derivative of $u(c_{1t})$ with respect to c_{1t}, and $d_{t+1} = X_{t+1} F_k \left(t+1 \right))$.

The first two optimality conditions are the Euler equations for bonds and physical assets, respectively. The last two optimality conditions are the intratemporel conditions on the labor market and intermediate good market, respectively.

Condition (4) states that if the collateral constraint is not binding ($\mu_t = 0$), the marginal benefit of borrowing to increase today's consumption is equal to the expected marginal cost of repaying back tomorrow. If the collateral constraint binds, the shadow price of relaxing the collateral constraint is positive ($\mu_t > 0$), so the marginal benefit of borrowing is greater than its expected marginal cost. Condition (5) states that the marginal cost of buying one additional unit of physical asset at price q_t is equal to its expected marginal benefit. If the collateral constraint is expected to bind, the marginal cost exceeds the marginal benefit by

$\mathbb{E}_t\left[\kappa_{t+1} q_{t+1} \mu_{t+1}\right]$.

Condition (6) states that the marginal productivity of labor demand is equal to the marginal disutility of labor supply plus the financing cost of labor from the working capital loan. The financing cost is higher when the collateral constraint binds. Condition (7) states that the marginal productivity of the intermediate input is equal to its price plus the financing cost of the intermediate input from the working capital loan. The financing cost of the intermediate input is higher when the collateral constraint binds.

2.2 Hand-to-mouth households' optimization problem

There is a continuum of identical "hand-to-mouth" households of measure 1. The preferences of a "hand-to-mouth" consumer indexed by 2 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{2t} - G(L_{2t})), \tag{8}$$

where C_{2t} is consumption, L_{2t} is labor supply, and $u(.)$ is the same utility function as in section 2.1. G(L) is a convex, strictly increasing, and continuously differentiable function that measures the disutility of labor. These preferences (known as GHH preferences due to Greenwood et al. (1988)) remove the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.The budget constraint of "hand-to-mouth" households is given by

$$P_t C_{2t} = P_t w_t L_{2t} + T_t. \tag{9}$$

The *hand-to-mouth* households chooses consumption and labor to maximize their utility (8) subject to their budget constraint (22), taking prices as given. Their optimality condition is given by

$$G'(L_{2t}) = w_t. \tag{10}$$

Condition (10) states that the marginal disutility of labor supply for *asset holder* consumers is equal to the real wage rate.

2.3 Final good producers

The final good producer combines the differentiated goods produced by retailers using a CES production technology. The retailers are indexed by $j \in [01]$.

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{11}$$

where $\varepsilon > 1$ is the elasticity of substitution between retailers' goods. The competitive final good producer chooses the demand for each differentiated good $y_{j,t}$ to maximizes his profit given by $P_t Y_t - \int_0^1 p_{j,t} y_{j,t} dj$. The optimization of final good producer' profit gives the iso-elastic demand curve faced by each retailers

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\frac{1}{\varepsilon}} Y_t, \tag{12}$$

where P_t is the standard price of the final good given by $P_t = \left(\int_0^1 y_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$.

2.4 Retailers with price-stickiness

There are monopolistically competitive differentiated good producing firms. Each retailer sets his price $p_{j,t}$ and faces a convex adjustment cost a la Rotemberg (1982), which is given by $\mathcal{A}_t = \frac{\theta}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 Y_t$. Each retailer j maximizes his present discounted value of profits taking as given the price P_t, aggregate output Y_t, the price of the intermediate good P_t^e, and the stochastic discount factor $\beta^t \mathcal{M}_t$.

$$\max_{p_{j,t}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathcal{M}_t \left\{ \left(\frac{p_{j,t}}{P_t} - \frac{P_t^e}{P_t} \right) y_{j,t} - \frac{\theta}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 Y_t \right\} \quad st \quad y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t. \tag{13}$$

The real marginal cost is represented by $\frac{P_t^e}{P_t}$. By taking the first order condition and using the symmetric price $p_{jt} = P_t$, I get the standard non-linear New Keynesian Phillips Curve (NKPC):

$$\pi_t (1 + \pi_t) = \frac{\varepsilon}{\theta} \left(X_t - \frac{\varepsilon - 1}{\varepsilon} \right) + \beta \mathbb{E}_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right], \tag{14}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation and $X_t = \frac{P_t^e}{P_t}$ is the inverse of the retailer's markup. Equation 14 states when retailers anticipate higher inflation in the future, they adjust up today their price to smooth the cost of adjustment. If price is fully flexible (i.e. $\theta = 0$), retailers always set prices for a constant markup, which depends only on the elasticity of substitution between retailers' goods. The constant markup is given by $\frac{\varepsilon}{\varepsilon-1}$. To achieve the constant markup, retailers set prices to equate current marginal revenue to current marginal cost. If price is fully rigid (i.e. $\theta \to \infty$) retailers set once and for all their prices to equate average marginal cost to average marginal revenue.

2.5 Competitive equilibrium

In this section, I define the competitive equilibrium and the main credit channel through which sudden stops arise in this type of framework. The aggregate resource of the economy is given by

$$c_t + \frac{b_{t+1}}{R_t} - b_t = F\left(1, L_t, v_t\right) - p_t^v v_t - \mathcal{A}_t, \tag{15}$$

where b_{t+1} is the real bond holdings, $c_t = c_{1t} + c_{2t}$ is aggregate consumption, the term $\frac{b_{t+1}}{R_t} - b_t$ represents the trade balance, and the term $F\left(1, L_t, v_t\right) - p_t^v v_t - \mathcal{A}_t$ represents GDP.

A competitive equilibrium in this model is a stochastic sequence $Q_t = \{C_{1t}, C_{2t}, L_t, L_{2t}, v_t, b_{t+1}\}_{t\geq 0}$, inflation and markup $\{\pi_t, X_t\}_{t\geq 0}$ and prices $P_t = \{q_t, w_t\}_{t\geq 0}$ such that:

1. Given P_t, Q_t solves households' and firms' problems;
2. w_t and q_t are determined competitively that is: $G'(L_t) = w_t$, and q_t solves equation (5);
3. the New Keynesian Phillips curve (NKPC) (14) holds:
4. markets clear:
   (a) labor market: $L_t = L_{2t}$,
   (b) capital market: $k_t = 1$,
   (c) aggregate resource: equation (15) is satisfied.

2.6 Monetary policy instrument

The central bank sets a domestic nominal interest rate i_t on domestic bond B_t^d to control the inflation rate. I assume that only asset holders have access to the domestic bond. With This assumption, Equation 3 holds at the equilibrium since. At the equilibrium $B_t^d = 0$. The no-arbitrage condition between domestic and foreign bond is given by:

$$\beta \mathbb{E}_t \left[R_t - \frac{(1+i_t)}{1+\pi_{t+1}} u'(t+1) \right] + \mu_t = 0, \tag{16}$$

where R_t is the foreign interest rate taking as given, i_t is the domestic nominal interest rate, and μ_t is the gain of relaxing the borrowing constraint on foreign asset. Note that only the foreign bond is subject to a collateral constraint. The no-arbitrage is the combination of the Euler equation of the foreign bond and the domestic bond. Equation 16 allows to recover back the optimal domestic nominal interest rate.

I assume that the central banks set the nominal in three different regimes, which are: inflation targeting using a the Taylor rule, discretionary monetary policy without capital control, and discretionary monetary policy with capital control. The Taylor rule on the nominal domestic interest rate is given by

$$1 + i_t = (1 + \bar{i}) \left(\frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_\pi}, \tag{17}$$

where $\bar{i}$ is the average net domestic nominal interest rate and $\bar{\pi}$ is the target inflation. In the baseline inflation targeting regime, I assume that the target inflation is 0 . The discretionary monetary policy with and without capital control is set via an optimal time-consistent problem presented in the following section.

3 Optimal Time-consistent Planner's Problem

In this section, I analyze the optimal time-consistent solution. The planner chooses the optimal current allocations taking as given the future policy functions. I study Two main regimes. The first is the discretionary monetary policy without capital and the second is the discretionary monetary policy with capital control. In the discretionary monetary policy with capital control, I choose to use a tax on foreign debt — a capital control — to decentralize the planner's solution and the no-arbitrage condition in equation 16 to get back the optimal domestic nominal interest rate. The taxes collected are redistributed in the form of lump-sum transfers to asset households. This section presents the optimization problem to answer two main questions. First, how effective is an optimal discretionary monetary policy to reduce both the severity and frequency of a financial crisis in a limited financial market participation economy? Second, what are the benefit of the joint design of monetary policy and capital control in a household heterogeneity environment?

Input wedge: In general, the production function is inefficient (different from the first best allocations) under an arbitrary monetary policy or capital control policy. Moving the equilibrium allocations from the first best can be conveniently summarized in the input wedge, defined below:

$$\varphi_t \equiv \frac{F_v(1, l_t, v_t)}{p_t^v} - \frac{F_l(1, l_t, v_t)}{G'(l_t)}, \tag{18}$$

where F_v F_l are the marginal product of imported input and the marginal product of labor respectively. The input wedge in equation 18 is defined as the difference between the relative benefit of labor the imported input and the relative benefit of labor. The relative benefit is the ratio of the value of employment to the cost of supplying labor. Note that in my framework the relevant wedge is the input wedge and not the labor wedge usually used in the literature [5] . The input wedge is the relevant wedge because of the presence of the two inputs in the production function, which are labor $l_t$and imported input v_t.

At a first-best allocation $\varphi_t = 0$ (see appendix B). A positive input wedge, $\varphi_t > 0$, reflects the relative benefit of the imported input exceed the relative benefit of labor. In this case, the economy experiences a recession. Conversely, a negative labor wedge, $\varphi_t < 0$, reflects relative benefit of the imported input is too low compared to the the relative benefit of labor . In this case, the economy experiences a boom.

Combining (6) and (7) and using the definition of the input wedge, the real marginal cost X_t of retailers satisfies the following equation.

$$\Big[\varphi_t + \frac{F_l(1, l_t, v_t)}{G'(l_t)}\Big] X_t = 1 - \frac{1}{\epsilon} + \phi \frac{\mu_t}{u'(c_{1t})}, \tag{19}$$

where $\frac{1}{\epsilon}$ represents wage subsidy to offset the monopolistic distortion.

Following Klein et al. (2008), Bianchi and Mendoza (2018) I focus on Markov stationary policy rules that are expressed as functions of the payoff-relevant state variables (b, s). A Markov perfect equilibrium is characterized by a fixed point in these policy rules, at which the policy rules of future planners that the current planner takes as given to solve its optimization problem match those that the current planner finds optimal to choose. Hence, the planner does not have the incentive to deviate from other planners' policy rules, thereby making these rules time consistent. Let $\mathcal{B}(b, s)$ be the policy functions for foreign bond holding of futures planners. Taking as given $\Big\{\mathcal{B}(b, s), \mathcal{C}_1(b, s),\ \mathcal{L}(b, s),\ \mathbf{v}(b, s),\ \boldsymbol{\mu}(b, s),\ \mathcal{Q}(b', s'),\ \boldsymbol{\pi}(b, s),\ \mathcal{X}(b', s')\Big\}$, the social planner solves problem 20 in the discretionary monetary policy without capital control regime. In the discretionary monetary policy with capital control regime, the social planner solves the same problem where the foreign bond Euler equation is not bind.

The foreign bond Euler equation implementability constraint has the multiplier $\gamma \geq 0$. The asset pricing implementability constraint has the multiplier $\xi \geq 0$. The "hand-to-mouth" consumer resource constraint has the multiplier δ. The firm price setting implementability constraint has the multiplier ϑ. The economy's resource constraint has the multiplier $\lambda \geq 0$. The collateral constraint has the multiplier $\mu^* \geq 0$. The asset holders' slackness condition has the multiplier $\varsigma \geq 0$.

Definition: The recursive constrained-efficient equilibrium is defined by the policy function b'(b, s) with associated decision rules $c_1(b, s),\ l(b, s),\ v(b, s),\ \mu(b, s)$, pricing function q(b, s), inflation and markup

[5]see Coulibaly (2018), Bianchi and Coulibaly (2021).

functions $\pi(b,s), X(b,s)$, value function $\mathcal{V}(b,s)$, the conjectured function characterizing the decision rule of future planners $\mathcal{B}(b,s)$ and the associated decision rules $\mathcal{C}_1(b,s),\ \mathcal{L}(b,s),\ \mathbf{v}(b,s), ,\ \boldsymbol{\mu}(b,s)$, asset prices $\mathcal{Q}(b,s)$, and price inflation and the inverse of markup $\boldsymbol{\pi}(b,s)$, X(b,s) such that these conditions hold:

1. Social planner optimizes: $\mathcal{V}(b,s)$ and the policy functions $\Big\{b'(b,s), c_1(b,s),\ l(b,s),\ v(b,s), \mu(b,s),\ q(b,s),$ $\pi(b,s),\ X(b,s)\Big\}$ solves the problem 20 given $\Big\{\mathcal{B}(b,s), \mathcal{C}_1(b,s),\ \mathcal{L}(b,s),\ \mathbf{v}(b,s), \boldsymbol{\mu}(b,s),\ \mathcal{Q}(b,s),\ \boldsymbol{\pi}(b,s), \mathcal{X}(b',s')\Big\}$
2. The policy functions are time consistent: The conjectured policy functions that represent optimal choices of future planners match the corresponding recursive functions that represent optimal plans of the current planner, which are: $b'(b,s) = \mathcal{B}(b,s),\ c_1(b,s) = \mathcal{C}_1(b,s),\ l(b,s) = \mathcal{L}(b,s),\ v(b,s) = \mathbf{v}(b,s),$ $\mu(b,s) = \boldsymbol{\mu}(b,s)$, $q(b,s) = \mathcal{Q}(b,s), \pi(b,s) = \boldsymbol{\pi}(b,s),\ X(b,s) = \mathcal{X}(b,s)$.

$$\begin{aligned}
\mathcal{V}(b,s) &= \max_{c_1,c_2,b',l,v,q,\pi,\mu} \Big\{u(c_1) + \omega u(c_2 - G(l) + \beta \mathbb{E}_{s',s}\mathcal{V}(b',s')\Big\} \\
u'(c_1) &= \beta R \mathbb{E}_{s',s}\left[u'(\mathcal{C}_1(b',s')\right] + \mu, \\
q u'(c_1) &= \beta \mathbb{E}_{s',s}\left[u'(b',s')\left[\mathcal{X}(b',s')F_k\left(1,\mathcal{L}(b',s'),\mathbf{v}(b',s')\right) + \mathcal{Q}(b',s')\right] + \kappa' \boldsymbol{\mu}(b',s')\mathcal{Q}(b',s')\right] \\
\Big[\varphi + \frac{F_l(1,l,v)}{G'(l)}\Big]X &= 1 + \phi \frac{\mu}{u'(c_1)} \\
c_2 &= G'(l)l + \boldsymbol{t} \qquad (20) \\
\pi\left(1+\pi\right) &= \frac{\varepsilon}{\theta}\left(X - \frac{\varepsilon - 1}{\varepsilon}\right) + \beta \mathbb{E}_{s',s}\left[\frac{u'(\mathcal{C}_1(b',s')F\left(1,\mathcal{L}(b',s'),\mathbf{v}(b',s')\right)}{u'(c_1)F\left(1,l,v\right)}\boldsymbol{\pi}(b',s')\left(1+\boldsymbol{\pi}(b',s')\right)\right] \\
c_1 + c_2 + \frac{b'}{R} - \frac{b}{1+\pi} &= \left(1 - \frac{1}{2}\theta\pi^2\right)F\left(1,l,v\right) - p^v v \\
\frac{b'}{R_t} - \phi p^v v &\geq -\kappa q \\
\mu\left(\frac{b'}{R} - \phi p^v v + \kappa q\right) &= 0. \quad \mu \geq 0
\end{aligned}$$

Let define some auxiliary variables. $\Omega(b',s') \equiv \beta \mathbb{E}_{s',s}\left[R u'(\mathcal{C}_1(b',s')\right]$,

$\Delta(b',s') \equiv \beta \mathbb{E}_{s',s}\left[u'(b',s')\left[\mathcal{X}(b',s')F_k\left(1,\mathcal{L}(b',s'),\mathbf{v}(b',s')\right) + \mathcal{Q}(b',s')\right] + \kappa' \boldsymbol{\mu}(b',s')\mathcal{Q}(b',s')\right]$, and

$$\Gamma((b',s',c_1,l,v,\mu) \equiv \frac{\varepsilon}{\theta}\left(X - \frac{\varepsilon-1}{\varepsilon}\right) + \beta \mathbb{E}_{s',s}\left[\frac{u'(\mathcal{C}_1(b',s')F\left(1,\mathcal{L}(b',s'),\mathbf{v}(b',s')\right)}{u'(c_1)F\left(1,l,v\right)}\boldsymbol{\pi}(b',s')\left(1+\boldsymbol{\pi}(b',s')\right)\right]$$

,

Lemma 3.1 *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w and τ_v be the wage and the imported input price subsidy respectively such that and $\tau_w = \tau_v = \frac{1}{\varepsilon}$. Then, in the absence of a credit friction (i.e., $\mu_t = 0$ for all t), the constraint-efficient flexible* *prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by* $\omega = \frac{u'(c_{1t})}{u'(c_{2t}-G(l_t))}$.

Proof: See appendix C.2.

The optimal relative weight ω implies that the resource constraint of "hand-to-mouth" consumer is not bind. It means that the social planner can optimally chooses the lump sum transfer t to replicate the competitive allocations of "hand-to-mouth" consumer. If the social planner do not have this instrument, lemma (3.1) says that the constraint-efficient flexible prices allocations won't coincide with the competitive equilibrium allocations. A number of reasons may make this optimal weight infeasible in reality. Consider a relative weight that implies that the social planner should tax hand-to-mouth to redistributed to asset holders. For political reasons, it may not be feasible.

3.1 Discretionary monetary policy with free capital flows

In this section I characterize the optimal monetary policy under discretion when there is no capital control. I also discuss how does inequality affect this optimal monetary policy. The optimal monetary policy under discretion solves problem 20. The first proposition characterizes the optimal monetary policy under discretion in absence of credit frictions and the second proposition generalizes the first one and conclude that there is exists a comprise between price stability, financial stability and inequality.

Proposition 1 *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let* τ_w *and* τ_v *be the wage and the imported input price subsidy respectively such that and* $\tau_w = \tau_v = \frac{1}{\varepsilon}$. *In the absence of credit friction* (*i.e.,* $\mu_t = 0$ *for all* t) *the optimal monetary policy under discretion strictly stabilizes inflation* (*i.e* $\pi_t = 0$ *for all* t) *and the optimal relative weight is given by* $\omega = \frac{u'(c_{1t})}{u'(c_{2t}-G(l_t))}$. *Further if at the equilibrium, when the relative weight* $\omega \neq \frac{u'(c_{1t})}{u'(c_{2t}-G(l_t))}$ *, the optimal monetary policy under discretion deviates from price stability* (*i.e* $\pi_t \neq 0$).

The proof is straightforward and comes from lemma (3.1). By setting always the inflation at its target, the social planner will replicate the flexible-price allocations. Lemma (3.1) establishes that the constraint-efficient flexible prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by $\omega = \frac{u'(c_{1t})}{u'(c_{2t}-G(l_t))}$. This ends the proof. It is important to understand that in the absence of household heterogeneity, it is well know in the New Keynesian literature that price stability is optimal in absence of credit friction. Central banks do not have any incentive to deviates from the price stability policy when the collateral constraint is not binding. The first part of the proposition says that, when it is possible for the central bank and the fiscal authority to coordinate and optimally choose the lump sum transfer, it is optimal for the central banks to not deviate from its target inflation. The second part of the proposition says whenever the coordination is not possible or the collateral binds, the central bank has an incentive to deviate from the target inflation.

Proposition 1 breakdowns what Blanchard and Galí (2007) call the divine coincidence observed in the standard New Keynesian models in the absence of credit friction. A central bank deviates from its price stability objective because of a concern of inequality. My result is consistent with Acharya et al. (2020) who find in a Heterogeneous Agents New Keynesian (HANK) framework, that a concern of inequality leads the monetary-maker to weight more an economic activity stabilisation than a price stabilization.

Proposition 2 *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let* τ_w *and* τ_v *be the wage and the imported input price subsidy respectively such that and* $\tau_w = \tau_v = \frac{1}{\varepsilon}$*. In the presence of credit friction, the optimal monetary policy under discretion is given by*

$$\underbrace{\theta \Phi y_t \pi_t}_{\text{Price stability motive}} = \tilde{w} u'(c_{1t}) + \underbrace{\left\{\sigma \frac{\kappa_t q_t}{c_{1t}} \tilde{w} - \phi \alpha p_t^v v_t \right\} \mu_t^*}_{\text{Ex-post financial stability motive}} + \underbrace{\sigma \frac{u'(c_{1t})}{c_{1t}} \tilde{w} \gamma_t}_{\text{Ex-ante financial stability motive}} + \underbrace{\eta G''(l) l^2 \delta_t}_{\text{Inequality motive}} \tag{21}$$

$$\text{where } \Phi = \Phi_0 + \beta \mathbb{E}_t \left[\Phi_1 \pi_{t+1}\right], \text{ with } \Phi_0 = \underbrace{\frac{\varepsilon}{\theta}\Big[- \alpha v_t \frac{F_{vv}(t)}{p_t^v} s_t z_t^{-2} + \eta l_t \frac{F_{vl}(t)}{p_t^v} s_t z_t^{-2} + \sigma \tilde{\omega} \frac{\phi \mu_t}{c_{1t}} z_t^{-1} \Big]}_{>0} \frac{\lambda_t}{1+2\pi_t},$$

$\delta_t = -\omega u'(c_{2t} - G(l_t)) + u'(c_{1t}) + \sigma \frac{u'(c_{1t})}{c_{1t}} \gamma_t + \sigma \frac{\kappa_t q_t}{c_{1t}} \mu_t^* - \theta \frac{\Gamma_c \lambda_t}{1+2\pi_t} \pi_t y_t$*, and* $\gamma_t = 0$ *if for all t* $\mu_t^* = 0$

Proof: See appendix C.3.

In Proposition 2 , the social planner's Lagrange multiplier on the collateral constraint μ_t^* captures the adjustment in the monetary policy when the economy is in crisis. The multiplier on the foreign bond Euler equation γ_t captures the adjustment in the monetary policy when the monetary authority anticipates a financial crisis in the future. The Lagrange multiplier on the resource constraint σ_t of the hand-to-mouth consumers captures the inequality motive. In the absence of this type of consumers, the multiplier $\sigma_t = 0$. Proposition 2 is a generalization of Proposition 1. In the absence of credit friction (i.e., $\mu_t^* = \gamma_t = 0$) and in the absence of household heterogeneity (i.e., $\delta_t = 0$), price stability will perfectly stabilize output with the given optimal relative weight. In the presence of credit friction and inequality,policymakers face a comprise between price stability, financial stability and inequality.

Ex-post financial motive: The second term on the right side of 21 captures the ex-post financial motive in the setting of a monetary policy. It implies that monetary authority has incentive to deviate from price stability when the collateral constraint binds. The degree to which the financial crisis affect the optimal monetary policy depends on two outcomes, which are a weighted value of the value of the collateral given by $\sigma\frac{\kappa_t q_t}{c_{1t}}\tilde{w}$ and a weighted value of the working capital loan $\phi\alpha p_t^v v_t$. It is quite intuitive to see why the sensitivity of the optimal monetary policy during the financial crisis depends on these two outcomes. First, it is worth noting that when the collateral constraint binds, the foreign loan is equal to the current value of the collateral minus the current value of the working capital loan, weighted by the foreign interest rate. Second, the monetary authority internalizes the fact that the borrowing decision affect the current asset price. So the monetary authority can affect the current value of the collateral and the working capital loan.

During the financial crisis whether the central banks should conduct a contractionary monetary policy or an expansionary monetary policy is ambiguous. In the absence of a working capital loan (i.e., $\phi = 0$), during the financial crisis, the central banks should conduct an expansionary monetary policy. By lowering domestic nominal interest during the crisis, investors demand lower premium on their domestic physical asset which raised the asset price and relax the collateral constraint.

Ex-ante financial motive: The third term on the right side of 21 captures the ex-ante financial motive in the setting of a monetary policy. It implies that there is a role for monetary policy as a macro-prudential tool.The monetary authority can 'lean against the wind' in advance of a financial crisis, when policy is made under discretion (absence of commitment). Departing from inflation stabilization may have a benefit even if the economy is not currently borrowing-constrained. Devereux et al. (2019)shows in a representative agent model that the monetary authority should not try to 'lean against the wind' in advance of a financial crisis, when policy is made under discretion because they use future-asset price as opposed to a current-asset price collateral constraint. In a flexible price framework without working capital loan Ottonello et al. (2021) show that the desirability of macroprudential policies critically depends on the specific form of collateral used in debt contracts. They argued that the equilibrium is inefficient when current prices affect collateral but there is no inefficiency when only future prices affect collateral.

In normal time (i.e., when the collateral does not bind), if the monetary authority anticipates financial crises in the future (i.e., γ_t positive), they are more likely to conduct an expansionary monetary policy since the coefficient on γ_t in 21 is positive. By lowering domestic nominal interest, it lowers the demand for foreign bond and reduce vulnerability to capital inflows in the future. Coulibaly (2018) finds similar result in two consumption goods model, that are tradable and non-tradable goods. He shows that a sufficient condition to conduct an expansionary monetary policy in normal time is when the intra-temporel elasticity of substitution is greater than the inter-temporel elasticity substitution.

Inequality motive: The presence of household heterogeneity distort the price stability in three dimensions. First, in the absence of credit friction, the central banks have an incentive to deviate from the price stability for inequality concern. Second, inequality amplifies the ex-ante financial motive response for monetary policy. The monetary policy should be more expansionary in normal time to mitigate the distributional impacts of the financial crisis. Third, inequality may affect qualitatively the ex-post financial motive response for monetary policy. The monetary policy is less likely to be contractionary during the financial crisis.

3.2 Discretionary monetary policy with capital control

In this section I characterizes the optimal monetary policy under discretion when capital flows are taxed. I also discuss how does inequality affect this optimal monetary policy. The optimal monetary policy under discretion solves problem 20 without the foreign bond Euler equation implementability constraint (i.e., the multiplier $\gamma_t = 0$ for all t).

Corollary 3.1.1 *In the presence of capital control, monetary policy should not be used as a macroprudential tool.*

In the presence of capital control, the foreign bond Euler equation implementability constraint is never always bind (i.e., the multiplier $\gamma_t = 0$ for all t). There is no ex-ante financial motive for the monetary policy. I conclude then that the monetary policy should not be used as a macroprudential tool. Capital control through a tax on foreign debt can efficiently act as macroprudential tool.

4 Conclusion

This paper studies the joint design of monetary policy and capital control in an environment with a motive for both financial stability and price stability. I build an equilibrium business cycle model with a current-price collateral constraint, household heterogeneity due to a limited financial market participation, and nominal rigidity. I show that, in the absence of credit friction (i.e., the collateral is never binding), the monetary authority under the discretionary monetary policy has an incentive to deviate implementing price stability (the divine coincidence does not hold). In addition, I show that in the case of financial instability due to credit frictions, the monetary authority under the discretionary monetary policy should adopt a prudential monetary policy only if capitals flows are free. This ex-ante prudential monetary policy is exacerbated by household inequality. In the absence of a working capital loan procyclical monetary policy is never optimal.

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Appendix

A Competitive equilibrium

The competitive equilibrium is summarized by the following equations

$$
\begin{aligned}
u'(t) &= \beta\mathbb{E}_t\left[\frac{R_t}{1+\pi_{t+1}}u'(t+1)\right]+\mu_t, \\
q_t u'(t) &= \beta\mathbb{E}_t\left[\left\{X_{t+1}F_k\left(1,L_{t+1},v_{t+1}\right)+q_{t+1}\right\}u'(t+1)+\kappa_{t+1}q_{t+1}\mu_{t+1}\right], \\
X_tF_l\left(1,L_t,v_t\right) &= G'(L_t), \\
X_tF_v\left(1,L_t,v_t\right) &= p_t^v+\phi\frac{\mu_t}{u'(t)}p_t^v, \\
c_{2t} &= G'(L_t)L_t+t_t. \\
\pi_t\left(1+\pi_t\right) &= \frac{\varepsilon}{\theta}\left(X_t-\frac{\varepsilon-1}{\varepsilon}\right)+\beta\mathbb{E}_t\left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\pi_{t+1}\left(1+\pi_{t+1}\right)\frac{Y_{t+1}}{Y_t}\right], \\
c_{1t}+c_{2t}+\frac{b_{t+1}}{R_t}-\frac{b_t}{1+\pi_t} &= F\left(1,L_t,v_t\right)-p_t^v v_t-\mathcal{A}_t, \\
\mu_t\left(\frac{b_{t+1}}{R_t}-\phi p_t^v v_t+\kappa_t q_t k_t\right) &= 0.\quad \mu_t\geq=0 \\
Taylor\ Rule\ regime\quad 1+i_t &= i\left(\frac{1+\pi_t}{1+\pi}\right)^{\phi_\pi} \\
\beta\mathbb{E}_t\left[R_t-\frac{(1+i_t)}{1+\pi_{t+1}}u'(t+1)\right]+\mu_t &= 0
\end{aligned}
$$

I solve for the competitive equilibrium in which price is fully stable (i.e $\pi_t=0$).

B First best allocation

The first best allocation solves problem (22)

$$
\max_{c_{1t},c_{2t},b_{t+1},l_t,v_t}\quad \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[u(c_{1t})+\omega u(c_{2t}-G(l))\Big], \tag{22}
$$

$$
s.t\quad c_{1t}+c_{2t}+\frac{b_{t+1}}{R_t}-b_t = F\left(1,l_t,v_t\right)-p_t^v v_t
$$

Let λ_t the multiplier on the aggregate resource constraint. The first best optimality conditions are given by

$$c_{1t} \quad :: \quad u'(c_{1t}) - \lambda_t = 0 \tag{23}$$

$$c_{2t} \quad :: \quad \omega u'(c_{2t} - G(l_t)) - \lambda_t = 0 \tag{24}$$

$$l_t \quad :: \quad -\omega G'(l_t) u'(c_{2t} - G(l_t)) + F_l\,(1, l_t, v_t \lambda_t = 0 \tag{25}$$

$$v_t \quad :: \quad \left[F_v\left(1, l_t, v_t - p_t^v\right]\lambda_t = 0 \tag{26}$$

$$b_{t+1} \quad :: \quad -\frac{1}{R_t}\lambda_t + \beta\lambda_{t+1} = 0 \tag{27}$$

Combining conditions (23)-(26) I obtain that the input wedge is zero at a first best allocation

$$\varphi_t \equiv \frac{F_v(1, l_t, v_t)}{p_t^v} - \frac{F_l(1, l_t, v_t)}{G'(l_t)} = 0$$

In addition the first best relative weight ω is equal to the relative marginal utility. That is: $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$

C Discretionary monetary policy

Under the discretionary monetary policy, the central banks solves the following problem. Let define some auxiliary variables. $\Omega(b', s') \equiv \beta \mathbb{E}_{s',s}\left[R u'(\mathcal{C}_1(b', s')\right]$, $X_t = \left[\varphi_t + \frac{F_l(1, l_t, v_t)}{G'(l_t)}\right]^{-1}\left\{1 - \frac{1}{\epsilon} + \phi \frac{\mu_t}{u'(c_{1t})}\right\}$,

$\Delta(b', s') \equiv \beta \mathbb{E}_{s',s}\left[u'(b', s')\left[\mathcal{X}(b', s') F_k\left(1, \mathcal{L}(b', s'), \mathbf{v}(b', s')\right) + \mathcal{Q}(b', s')\right] + \kappa' \boldsymbol{\mu}(b', s') \mathcal{Q}(b', s')\right]$, and

$\Gamma((b', s', c_1, l, v, \mu) \equiv \frac{\varepsilon}{\theta}\left(\phi \frac{\mu_t}{u'(c_{1t})\varphi_t} - \frac{\varepsilon - 1}{\varepsilon}\right) + \beta \mathbb{E}_{s',s}\left[\frac{u'(\mathcal{C}_1(b', s') F\left(1, \mathcal{L}(b', s'), \mathbf{v}(b', s')\right)}{u'(c_1) F\left(1, l, v\right)} \boldsymbol{\pi}(b', s')\left(1 + \boldsymbol{\pi}(b', s')\right)\right]$

,

$$\mathcal{V}(b, s) = \max_{c_1, c_2, b', l, v, q, \pi, \mu}\left\{u(c_1) + \omega u(c_2 - G(l) + \beta \mathbb{E}_{s',s} \mathcal{V}(b', s')\right\} \tag{28}$$

$$u'(c_1) = \Omega(b', s') + \mu, \qquad : \gamma_t \tag{29}$$

$$q u'(c_1) = \Delta(b', s') \qquad : \xi_t \tag{30}$$

$$c_2 = G'(l) l + \boldsymbol{t} \qquad : \delta_t \tag{31}$$

$$\pi\,(1 + \pi) = \Gamma((b', s', c_1, l, v) \qquad : \vartheta_t \tag{32}$$

$$c_1 + c_2 + \frac{b'}{R} - b = \left(1 - \frac{1}{2}\theta \pi^2\right) F\,(1, l, v) - p^v v \qquad : \lambda_t \tag{33}$$

$$\frac{b'}{R_t} - \phi p^v v \geq -\kappa q \qquad : \mu_t^* \tag{34}$$

$$\mu\left(\frac{b'}{R} - \phi p^v v + \kappa q\right) = 0. \qquad : \varsigma_t \tag{35}$$

The social planner's optimality conditions are given by

$$c_{1t} \quad :: \quad u'(c_{1t}) - \gamma_t u''(c_{1t}) - \xi_t q_t u''(c_{1t}) + \vartheta_t \Gamma_3(t+1) - \lambda_t = 0 \tag{36}$$

$$c_{2t} \quad :: \quad \omega u'(c_{2t} - G(l_t)) - \delta_t - \lambda_t = 0 \tag{37}$$

$$b_{t+1} \quad :: \quad \beta \mathbb{E}_{s',s} \mathcal{V}_b(b', s') + \gamma_t \Omega_1(t+1) + \xi_t \Delta_1(t+1) + \vartheta_t \Gamma_1(t+1) - \frac{1}{R}\lambda_t + \frac{1}{R}\mu_t^* + \frac{1}{R}\varsigma_t \mu_t = 0 \tag{38}$$

$$l_t \quad :: \quad -\omega G'(l_t) u'(c_{2t} - G(l_t)) + \delta_t \left(G''(l)l + G'(l)\right) + \vartheta_t \Gamma_4 + \lambda_t (1 - \frac{1}{2}\theta \pi_t^2) F_l\left(1, l, v\right) = 0 \tag{39}$$

$$v_t \quad :: \quad \vartheta_t \Gamma_5(t+1) + \lambda_t \Big\{ \left(1 - \frac{1}{2}\theta \pi_t^2\right) F_v\left(1, l, v\right) - p_t^v \Big\} - \phi \mu_t^* p_t^v - \phi p_t^v \varsigma_t \mu_t = 0 \tag{40}$$

$$q_t \quad :: \quad -\xi_t u'(c_{1t}) + \kappa_t \mu_t^* + \kappa_t \varsigma_t \mu_t = 0 \tag{41}$$

$$\mu_t \quad :: \quad \gamma_t + \varsigma_t \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t \right) + \vartheta_t \Gamma_6(t+1) = 0 \tag{42}$$

$$\pi_t \quad :: \quad -(1 + 2\pi_t)\vartheta_t - \theta \pi_t F\left(1, l, v\right) \lambda_t = 0 \tag{43}$$

$$KT \quad :: \quad \mu_t^* \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t \right) = 0 \tag{44}$$

$$EC \quad :: \quad \mathcal{V}_b(b_t, s_t) = \lambda_t \tag{45}$$

C.1 Lemma 1

Lemma C.1 *It is optimal to set $\varsigma_t \mu_t = 0$ for all t.*

Proof: Suppose $\mu_t * > 0$, by the KT condition in equation (44) $\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t = 0$. Then $\mu_t \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t \right)$ is equal to zero. So Condition (35) is satisfied. It is then optimal to set $\varsigma_t = 0$. Suppose now that $\mu_t * = 0$, by the KT condition in equation (44) $\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t > 0$. Then $\mu_t = 0$.

C.2 Proof of lemma 3.1

Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w and τ_v be the wage and the imported input price subsidy respectively such that and $\tau_w = \tau_v = \frac{1}{\varepsilon}$. Then, in the absence of a credit friction, the constraint-efficient flexible prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$.

Proof: Under flexible prices, Condition (43) shows that the multiplier $\vartheta_t = 0$. In addition, in the absence of credit friction, $\mu_t * = \mu = 0$, which implies from condition (41) that the multiplier $\xi = 0$. Suppose that the bond Euler equation implementability constraint is not bind that is $\gamma_t = 0$.

Combining (36), (38), and (45) gives the foreign bond Euler equation $u'(c_{1t}) = \beta R_t \mathbb{E}_{s',s} u'(c_{1t+1})$. Setting $\gamma_t = 0$, is then optimal. In addition, from condition it is optimal to set the multiplier $\nu_t = 0$ so that $F_v(1, l_t, v_t) - p_t^v = 0$. Now setting the relative weight $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$ leads $\delta_t = 0$. Given that, condition (39) shows that $F_l(1, l_t, v_t) = G'(l_t)$.

C.3 Proof of proposition 2

The proof uses the social planner's optimality conditions under discretionary monetary policy. I combine conditions (37) in (39)

$$-\lambda_t G'(l) + \delta_t G''(l) l + \vartheta_t \Gamma_4(t+1) + \lambda_t \left(1 - \frac{1}{2}\theta\pi_t^2\right) F_l(1, l, v) = 0 \tag{46}$$

I rearrange (46) and (40) to get

$$\alpha\lambda_t \left(1 - \frac{1}{2}\theta\pi_t^2\right) F = \lambda_t G'(l) l_t - \delta_t G''(l) l^2 - \vartheta_t \Gamma_4 l_t \tag{47}$$

$$\eta\lambda_t \Big\{ \left(1 - \frac{1}{2}\theta\pi_t^2\right) F \Big\} = -\vartheta_t \Gamma_5 v_t + \phi \mu_t^* p_t^v v_t + \lambda_t p_t^v v_t \tag{48}$$

I then substitute (47) into (48) to obtain

$$-\vartheta_t(\alpha\Gamma_5 v_t - \eta\Gamma_4 l_t) + \phi\alpha\mu_t^* p_t^v v_t + \Big\{\alpha p_t^v v_t - \eta l_t G'(l)\Big\}\lambda_t + \eta\delta_t G''(l) l^2 = 0 \tag{49}$$

Let $\tilde{w} \equiv \eta l_t G'(l) - \alpha p_t^v v_t$. Now, I use (46) to eliminate the lagrange multiple λ_t in (49), which gives

$$-\vartheta_t(\alpha\Gamma_5 v_t - \eta\Gamma_4 l_t + \tilde{w}\Gamma_3) + \phi\alpha\mu_t^* p_t^v v_t - \tilde{w} u'(c_{1t}) + \tilde{w} u''(c_{1t})\gamma_t + \xi_t \tilde{w} q_t u''(c_{1t}) + \eta\delta_t G''(l) l^2 = 0 \tag{50}$$

I use conditions (36) and (37) to obtain $\delta_t = \omega u'(c_{2t} - G(l_t)) - u'(c_{1t}) + \gamma_t u''(c_{1t}) + \xi_t q_t u''(c_{1t}) - \vartheta_t \Gamma_3(t+1)$. Condition (41) gives $\xi_t = \frac{\kappa_t}{u'(c_{1t})}\mu_t^*$ and condition (43) $\vartheta_t = -\theta \frac{\lambda_t}{1+2\pi_t} \pi_t F(1, l, v)$.

Let $\Phi \equiv (\alpha\Gamma_5 v_t - \eta\Gamma_4 l_t + \tilde{w}\Gamma_3)\frac{\lambda_t}{1+2\pi_t}$. I finally substitute those expressions into (50) to get

$$\theta\Phi y_t \pi_t = \tilde{w} u'(c_{1t}) - \Big\{\phi\alpha p_t^v v_t + \frac{\kappa_t}{u'(c_{1t})} \tilde{w} q_t u''(c_{1t})\Big\}\mu_t^* - \tilde{w} u''(c_{1t})\gamma_t - \eta G''(l) l^2 \delta_t \tag{51}$$

It can be shown using conditions (36) and (38) that γ_t is equal to zero if $\mu_t^* = 0$ for all t. Further, if the collateral is expected to bind in the future $\gamma_t \neq 0$. The multiplier γ_t captures the prudential motives for the discretionary monetary policy.

Now let denote $\sigma \equiv -\frac{u''(c_{1t})c_{1t}}{u'(c_{1t})}$ the risk aversion or the inverse of the elasticity of intertemporel of substitution. let $s_t \equiv 1 - \frac{1}{\epsilon} + \phi\frac{\mu_t}{u'(c_{1t})}$ and $z_t \equiv \varphi_t + \frac{F_l(1,l_t,v_t)}{G'(l_t)}$ so $X_t = z_t^{-1}s_t$. I can rewrite Φ to obtain

$$\Phi = \Phi_0 + \beta\mathbb{E}_t\left[\Phi_1\pi_{t+1}\right]$$

where $\Phi_0 = \underbrace{\frac{\varepsilon}{\theta}\Big[-\alpha v_t\frac{F_{vv}(t)}{p_t^v}s_t z_t^{-2} + \eta l_t\frac{F_{vl}(t)}{p_t^v}s_t z_t^{-2} + \sigma\tilde{\omega}\frac{\phi\mu_t}{c_{1t}}z_t^{-1}\Big]}_{>0}\frac{\lambda_t}{1+2\pi_t}$ and

$$\Phi_1 = \sigma\frac{\tilde{\omega}}{c_{1t}}\frac{u'(c_{1t+1})}{u'(c_{1t})}\frac{y_{t+1}}{y_t}\frac{(1+\pi_{t+1})\lambda_t}{1+2\pi_t}$$

Finally the optimal monetary policy under discretion satisfies:

$$\theta\Phi y_t\pi_t = \tilde{w}u'(c_{1t}) + \Big\{\sigma\frac{\kappa_t q_t}{c_{1t}}\tilde{w} - \phi\alpha p_t^v v_t\Big\}\mu_t^* + \sigma\frac{u'(c_{1t})}{c_{1t}}\tilde{w}\gamma_t - \eta G''(l)l^2\delta_t \tag{52}$$