Monetary policy, sticky wages and household heterogeneity

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Introduction

- Aggregate fluctuations matter for monetary policy
  - How does monetary policy (e.g. change in interest rate) affect economic activity?
  - Need good framework to tackle quantitatively those questions
  - Most of the central banks use New Keynesian framework with representative household
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- Heterogenous-Agents New Keynesian (HANK) models have gained prominence in recent years.
  - Werning (2015); Kaplan et al. (2018); Acharya and Dogra (2018).
  - A special case of HANK: Two-Agents New Keynesian (TANK), more tractable.
- Does TANK approximate HANK in terms of aggregate fluctuation to an aggregate shock?
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- Yes if prices sticky. (Debortoli and Gali (2018))
This paper

Does TANK approximate HANK in terms of aggregate fluctuation to an aggregate shock?

- Yes if prices sticky. (Debortoli and Gali (2018))
- No if wages are sticky.
Why sticky wages?

- Wages are sticky as prices in data: Taylor (1999); Nakamura and Steinson (2006); Dickens et al. (2007).

- Sticky wages help to generate addition persistent in MP shock, consistent with data: Christiano et al. (2005).
Contribution

- If wages are sticky, TANK does not approximate HANK.

- Suppose wages are sticky,
  - if prices are flexible, the aggregate behavior of a TANK model coincides with the behavior of the Representative-Agent New Keynesian (RANK).
  - consumption inequality wedge positively related to real price markup.
  - real price markup gap is zero under flexible price

- if prices are sticky, the initial aggregate response of output in TANK is 75% of the response in HANK
Summary

sticky prices: \( RANK \not\approx TANK \approx HANK \)

sticky wages: \( TANK \not\approx HANK \)

sticky prices and wages: \( TANK \not\approx HANK \)
Summary

sticky prices: $RANK \not\approx TANK \approx HANK$

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Summary

*sticky prices:* $\text{RANK} \not\approx \text{TANK} \approx \text{HANK}$

*sticky wages:* $\text{RANK} = \text{TANK} \not\approx \text{HANK}$

*sticky prices and wages:* $\text{RANK} \approx \text{TANK} \not\approx \text{HANK}$
Literature Review

- HANK under sticky prices:
  - Transmission mechanism of monetary policy: Auclert (2017); Kaplan et al. (18).
  - Welfare analysis: Bayer et al. (2015); Gornemann et al. (2016).
  - Analytical difference with RANK: Werning (2015); Bilbiie (2017), Acharya and Dogra (2018); Debortoli and Gali (2018),
  - This paper adds sticky wages.
  - Hagedorn et al. (2019a, 2019b).
  - This paper presents details comparison between TANK and HANK

- TANK under sticky prices and sticky wages:
  - Bilbiie (2008) (with sticky prices); Colciago (2011); Ascari et al. (2011).
  - This paper examines the case where production function is non linear and both agents are different at the steady state.
Model Setup

- HANK framework
  - Baseline New Keynesian framework with a continuum of households facing labor income risk.
  - Endogenous fraction of households are constrained in equilibrium.
  - Sticky wages.

- TANK framework
  - Baseline New Keynesian framework with two types of agents.
  - Exogenous fraction of households are constrained.
  - Sticky wages.
HANK framework

- **Households**
  - infinitely lived Households \( t=0,1,2,...\infty \).
  - consume and save into two assets: liquid asset \( B_{it} \), share of the equity fund \( F_{it} \) with price \( Q_{it} \).
  - Households face uninsurable labor income risk \( e_{it} \) following a markov process.
  - Household face an exogenous borrowing limit.
  - \( 1 - \delta \) firm’s profit goes to share holder.
  - \( \delta \) firm’s profit is shared between household according to a specific rule.

- **Wage union**
  - Imperfect competition on the labor market.
  - Middleman sets wage for every household by maximizing the net aggregate benefit for \( N_t \) unit of labor.
  - Middleman faces sticky wages.

- **Firms**
  - Imperfect competition on the good market.
  - monopolistic competitive firm faces sticky prices.
Household problem

\[
\max_{C_{it}, B_{i,t}} \ E \sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it})
\]

\[
C_{it} + Q_t F_t + \frac{B_{i,t}}{P_t} = \frac{B_{it-1}(1 + i_{t-1})}{P_t} + w_{it} N_{it} e_{it} + [Q_t + (1 - \delta)D_t] F_{it-1} + T_{it} - \Theta_{it}
\]

\[
\frac{B_{i,t}}{P_t} \geq -\Psi Y
\]

- \( e_{it} \): uninsurable labor income risk following AR(1) markov process.

- \( \Theta_{it} = e_{it} \theta_w \left( \frac{W_{it}}{W_{it-1}} - 1 \right)^2 Z_t \): Wage adjustment cost, \( Z_t \) aggregate output or labor.

- Borrowing limit.
Assumption: \( A_{it} = Q_tF_t + \frac{B_{i,t}}{P_t} \) is household net worth and \( v_t \in [0, 1] \). I assume as in DG (2018): \( Q_tF_{it} = \max[0, v_t A_{it}] \)

- Intuition: Only household with positive net worth claim firm’s profit (No short selling: \( F_{it} \geq 0 \)).
- Under the Assumption: \( F_{it} = \frac{A_{it}^+}{A_t^+} \) with \( A_{it}^+ = \max[0, A_{it}] \) and \( A_t^+ = \int_0^1 A_{it}^+ \, di \).
- Only the asset \( A_{it} \) relevant.
Transfer is assumed to follow the following rule:

\[ T_{it} = \left[ 1 + \tau_t^a \left( \frac{A_{it}^+}{A_t^+} - 1 \right) + \tau_t^e (e_{it} - 1) \right] \delta D_t \]

- Wealth-based rule (W-rule): \( \tau_t^a = 1 \) and \( \tau_t^e = 0 \); all profit goes to asset holder.
Wage setting problem

Following Hagedorn et al. (2019), I assume there exist a middleman chooses $\hat{W}_t = W_{it}$ by solving:

$$\max_{\hat{W}_t} \sum_{t=0}^{\infty} \beta^t \left[ \int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(W_t, \hat{W}_{t-1}, Z_t) di - \int_0^1 \frac{g(\hat{N}_t(W_t, W_t, Z_t))}{u'(C_t)} di \right]$$

$$s.t \quad \hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[ \frac{W_t}{\hat{W}_t} \right]^{\epsilon_w} N_t$$

- $g(.)$ is labor disutility.
- In red, aggregate benefit of labor per aggregate marginal utility.
- In blue, aggregate cost of labor per aggregate marginal utility.
- 2 is optimal labor demand from Employment agencies problem. (details)
Equilibrium conditions

\[ C_{it}^{-\sigma} \geq \beta (1 + r_t) \mathbb{E}(C_{it+1}^{-\sigma}) \]  \tag{3}

\[ \frac{\theta_p}{\varepsilon} \Pi^p_t (\Pi^p_t - 1) = \left[ m_t - \frac{\varepsilon - 1}{\varepsilon} \right] + \frac{\theta_p}{\varepsilon} \beta \mathbb{E}_t \left[ \Lambda_{t,t+1}\Pi^p_{t+1} (\Pi^p_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right] \]  \tag{4}

\[ \theta_w \Pi^w_t (\Pi^w_t - 1) = w_t (1 - \varepsilon_w) + \varepsilon_w N^w_t C_t^\sigma + \beta \theta_w \mathbb{E}_t \left[ \Pi^{w}_{t+1} (\Pi^{w}_{t+1} - 1) \frac{Z_{t+1}}{Z_t} \right] \]  \tag{5}

- 3 is standard Euler Equation which hold with equality for unconstrained agents; 4 is NK Price Phillips Curve (from standard firm problem) and 5 is NK Wage Phillips Curve.

- Three equations + Taylor rule + exogenous shock process of the model summary the economy.

- Why can’t solve this analytically: No possible aggregation for 3 but possible in TANK
TANK framework

- Time-invariant unconstrained agents of measures $1 - \lambda$ and time-invariant constrained agents of measures $\lambda$.
- $e_{it} = 1$ for every $i$ at every $t$.
- $F_{it} = \frac{1}{1-\lambda}$ only for unconstrained agents.
- The standard Euler Equation holds with equality for unconstrained agents.
- Constrained agents doesn’t participate on the bond market.
- At the equilibrium. $\frac{B_{i,t}}{P_t} = 0$
TANK: Characterization of the equilibrium

Proposition 1: Under sticky wages and sticky prices the following system of 4 equations summarizes the equilibrium.

\[
\begin{align*}
\pi^p_t &= \beta E \pi^p_{t+1} + \lambda_p \tilde{w}_t + k_p \tilde{y}_t & \text{Price NKPC} \\
\pi^w_t &= \beta E \pi^w_{t+1} - \lambda_w \tilde{w}_t + k_w \tilde{y}_t & \text{Wage NKPC} \\
\tilde{y}_t &= E \tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi^3)} \hat{r}_t + \frac{\Psi^2}{1+\Psi^3} E [\tilde{w}_{t+1} - \tilde{w}_t] & \text{DIS} \\
i_t &= \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t & \text{Taylor rule}
\end{align*}
\]

- There are two differences with the standard “3 equations” model:
  - there is one Phillips curve for each source of nominal rigidity
  - The aggregation only alters the demand side of the model: the Dynamic IS equation and the change is proportional to the change in the wage gap.
Lemma 1: Two measures are necessary and sufficient to aggregate the consumption in the economy. The two measures are:

- The consumption of unconstrained agents.
- A measure of consumption inequality.
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Proof. By definition, the aggregate consumption is: $C_t = (1 - \lambda)C^U_t + \lambda C^K_t$ or equivalently $C_t = C^U_t(1 - \lambda \gamma_t)$, where $\gamma_t = \frac{C^U_t - C^K_t}{C^U_t}$. Linearize around the steady state gives:

$\hat{c}_t = \hat{c}^U_t - \frac{\lambda}{1 - \lambda \gamma} \hat{\gamma}_t$
Lemma 1: Two measures are necessary and sufficient to aggregate the consumption in the economy. The two measures are:

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Proof. By definition, the aggregate consumption is: \( C_t = (1 - \lambda)C_t^U + \lambda C_t^K \) or equivalently \( C_t = C_t^U(1 - \lambda \gamma_t) \), where \( \gamma_t = \frac{C_t^U - C_t^K}{C_t^U} \). Linearize around the steady state gives:

\[
\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1 - \lambda \gamma_t} \hat{\gamma}_t
\]

The dynamics \( \hat{c}_t^U \) is known using the Euler equation for unconstrained agents. Next lemma characterizes the change in consumption inequality \( \hat{\gamma}_t \).
Lemma 2: The change in consumption inequality around the steady state is proportional to the percentage change in real price markup around the steady state.
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Proof.

- At the equilibrium, \( C_t^U - C_t^K = D_t \left( \frac{1-(1-\tau)\delta}{1-\lambda} \right) \), Where \( D_t \) is firm’s profit.

- The profit \( D_t = Y_t - w_tN_t - AC_t = \left[ (1 - \tilde{A}C_t) - (1 - \alpha) m_t \right] Y_t \) where \( m_t = \frac{w_t}{MPN} \) is the inverse of the real price markup.

- The markup determines the profit which determine the consumption inequality.

- Up to first order approximation, \( \hat{\gamma}_t = -\Psi_1 \hat{\mu}_t^p \), where \( \Psi_1 < 0 \) and \( \hat{\mu}_t^p \) is the real price markup deviation from its steady value \( \mu^p = \frac{e_p}{e_p - 1} \)

- In addition using the definition of the price markup: \( \hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha} \tilde{y}_t \)
- Under lemma 1 and lemma 2, the aggregate cons. is \( \hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma} \Psi_1 \left[ \hat{w}_t + \frac{\alpha}{1-\alpha} \hat{y}_t \right] \)

- Using the Euler equation for unconstrained agents, we have \( \hat{c}_t^U = \mathbb{E}\hat{c}_{t+1}^U - \frac{1}{\sigma} \hat{r}_t \)

- Using the aggregate resource constraint, up to first-order approximation the percentage change in aggregate consumption around the steady state is equal the percentage change in output around the steady state. That is \( \hat{c}_t = \hat{y}_t \)

- By definition, \( \hat{y}_t = \tilde{y}_t + \hat{y}_t^n \), where \( \hat{y}_t^n = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)} a_t \). For a monetary policy shock \( \hat{y}_t^n = 0 \)

- By combining the above points, we obtain the Dynamic IS equation shown in proposition 1.
Proposition 2: Under sticky wages (and flexible prices), the DIS equation
\[ \tilde{y}_t = E \tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi_3)} \hat{r}_t + \frac{\Psi_2}{1+\Psi_3} E [\tilde{w}_{t+1} - \tilde{w}_t] \] is reduced to
\[ \tilde{y}_t = E \tilde{y}_{t+1} - \frac{1}{\sigma} \hat{r}_t. \]
Proposition 2: Under sticky wages (and flexible prices), the DIS equation
\[ \tilde{y}_t = E\tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi_3)} \hat{r}_t^b + \frac{\Psi_2}{1+\Psi_3} E [\tilde{w}_{t+1} - \tilde{w}_t] \]
is reduced to
\[ \tilde{y}_t = E\tilde{y}_{t+1} - \frac{1}{\sigma} \hat{r}_t^b. \]

- Hence TANK is equivalent to RANK.

- Intuition: the change in consumption inequality is the key driving the difference between RANK and TANK.

- Only the percentage change in real price markup determines the change in consumption inequality.

- under flexible prices, the percentage change in real price markup is 0 (lemma 1)
What do we know at this stage

- Under sticky wages and flexible prices, TANK is equivalent to RANK.

- Have the equations that summarize the HANK equilibrium: but can not solve it analytically.

- Next step: Use numerical method to solve for the HANK equilibrium.
  - Compare HANK VS TANK under sticky prices and sticky wages.
HANK: Numerical solution

- Step 1: Solve for stationary distribution
  - I use Endogenous Grid point Method (EGM).

- Step 2: Solve for the aggregate fluctuation
  - Schmitt-Grohé and Uribe (2004); Bayer et al. (2019)
  - Solve a full system of 1769 equations of form:

\[ \mathbb{E} [X_t, X_{t+1}, Y_t, Y_{t+1}] = 0. \]
Aggregate fluctuations: IRF of MP shock
Aggregate fluctuations: Cumulative response, MP shock
Conclusion

- Under sticky prices and sticky wages, a Two-Agents New Keynesian (TANK) model cannot approximate Heterogeneous-Agents New Keynesian (HANK) model.

- The presence of sticky wages limits the role of Hand to Mouth in TANK.
Transfer rule

Transfer is assumed to follow the following rule:

\[ T_{it} = \left[ 1 + \tau_t^a \left( \frac{A_{it}^+}{A_{it}} - 1 \right) + \tau_t^e (e_{it} - 1) \right] \delta D_t \]

- Wealth-based rule (W-rule): \( \tau_t^a = 1 \) and \( \tau_t^e = 0 \); all profit goes to asset holder.

- Productivity-based rule (P-rule): \( \tau_t^a = 0 \) and \( \tau_t^e = 1 \); profit is shared proportional to household labor income risk.

- Uniform (U-rule): \( \tau_t^a = 0 \) and \( \tau_t^e = 0 \); lump sum transfer same for every household.
Employment agencies problem
Following Erceg et al. (1999)

\[
N_t = \left[ \int_0^1 e_{it} (N_{it})^{1-\frac{1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w-1}}
\]

Where \( \epsilon_w \) is the elasticity of substitution across labor services

\[
\max_{N_{it}} W_t N_t = \int_0^1 W_{it} N_{it} e_{it} \\
\text{s.t} \quad N_t = \left[ \int_0^1 e_{it} (N_{it})^{1-\frac{1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w-1}}
\]

The solution (the demand for the i-th consumer’s labor) is:

\[
N_{it} = \left[ \frac{W_t}{W_{it}} \right]^{\frac{\epsilon_w}{\epsilon_w-1}} N_t
\]
MP shock: RANK

Lambda = 1e-10  Alpha = 0

Output gap

Interest rate arm

Real wage

Inflation arm

Wage inflation

Monetary shock

back to Prop1
MP shock: TANK

Lambda = 0.21, Alpha = 0

Output gap

Interest rate

Real wage

Inflation

Wage inflation

Monetary shock

back to Prop1
Solution for stationary distribution
## Calibration

Debortoli and Gali (2018)

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My calibration

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back to step