Monetary policy, sticky wages and household heterogeneity

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Introduction

- Aggregate fluctuations matter for monetary policy
    - How does monetary policy (e.g. change in interest rate) affect economic activity?
    - Need good framework to tackle quantitatively those questions
    - Most of the central banks use New Keynesian framework with representative household

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- Heterogenous-Agents New Keynesian (HANK) models have gained prominence in recent years.
  - Werning (2015); Kaplan et al. (2018); Acharya and Dogra (2018).
  - A special case of HANK: Two-Agents New Keynesian (TANK), more tractable.

This paper

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  - Yes if prices sticky. (Debortoli and Gali (2018))
  - No if wages are sticky.

Why sticky wages?

- Wages are sticky as prices in data: Taylor (1999); Nakamura and Steinson (2006); Dickens et al. (2007).
- Sticky wages help to generate addition persistent in MP shock, consistent with data: Christiano et al. (2005).

Contribution

- If wages are sticky, TANK does not approximate HANK.
- Suppose wages are sticky,
  - if prices are flexible, the aggregate behavior of a TANK model coincides with the behavior of the Representative-Agent New Keynesian (RANK).
    - consumption inequality wedge positively related to real price markup.
    - real price markup gap is zero under flexible price
  - if prices are sticky, the initial aggregate response of output in TANK is 75% of the response in HANK

Summary

$$\begin{aligned} \textit{sticky prices}: &\quad \textit{RANK} \not\approx \textit{TANK} \approx \textit{HANK} \\ \textit{sticky wages}: &\quad \textit{TANK} \not\approx \textit{HANK} \\ \textit{sticky prices and wages}: &\quad \textit{TANK} \not\approx \textit{HANK} \end{aligned}$$

Summary

$$\begin{aligned}
\textit{sticky prices}: &\quad \textit{RANK} \not\approx \textit{TANK} \approx \textit{HANK} \\
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\end{aligned}$$

Literature Review

- HANK under sticky prices:
  - Transmission mechanism of monetary policy: Auclert (2017); Kaplan et al. (18).
  - Welfare analysis: Bayer et al. (2015); Gornemann et al. (2016).
  - Analytical difference with RANK: Werning (2015); Bilbiie (2017), Acharya and Dogra (2018); Debortoli and Gali (2018),
  - Optimal monetary policy: Bilbiie (2018).
  - This paper adds sticky wages .
  - Hagedorn et al. (2019a, 2019b).
  - This paper presents details comparison between TANK and HANK
- TANK under sticky prices and sticky wages:
  - Bilbiie (2008) (with sticky prices); Colciago (2011); Ascari et al. (2011).
  - This paper examines the case where production function is non linear and both agents are different at the steady state.

Model Setup

- HANK framework
  - Baseline New Keynesian framework with a continium of households facing labor income risk.
  - Endogenous fraction of households are constrained in equilibrium.
  - Sticky wages.
- TANK framework
  - Baseline New Keynesian framework with two types of agents.
  - Exogenous fraction of households are constrained.
  - Sticky wages.

HANK framework

- Households
  - infinitely lived Households t=0,1,2....∞.
  - consume and save into two assets: liquid asset B_{it}, share of the equity fund F_{it} with price Q_{it}.
  - Households face uninsurable labor income risk e_{it} following a markov process.
  - Household face an exogenous borrowing limit.
  - $1-\delta$ firm's profit goes to share holder.
  - δ firm's profit is shared between household according to a specific rule.
- Wage union
  - Imperfect competition on the labor market.
  - Middleman sets wage for every household by maximizing the net aggregate benefit for N_t unit of labor.
  - Middleman faces sticky wages.
- Firms
  - Imperfect competition on the good market.
  - monopolistic competitive firm faces sticky prices.

Household problem

$$\max_{C_{it}, \frac{B_{i,t}}{P_t}} \quad \mathbb{E} \quad \sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it})$$

$$C_{it} + Q_t F_t + \frac{B_{i,t}}{P_t} \quad = \quad \frac{B_{it-1}(1 + i_{t-1})}{P_t} + w_{it} N_{it} e_{it} + [Q_t + (1-\delta) D_t] F_{it-1} + T_{it} - \Theta_{it}$$

$$\frac{B_{i,t}}{P_t} \quad \geq \quad -\Psi Y$$

- e_{it}: uninsurable labor income risk following AR(1) markov process.
- $\Theta_{it} = e_{it} \frac{\theta_w}{2} \left(\frac{W_{it}}{W_{it-1}} - 1 \right)^2 \mathbb{Z}_t$:Wage adjustment cost, $\mathbb{Z}_t$ aggregate output or labor.
- Borrowing limit.

Household problem

Assumption : $A_{it} = Q_t F_t + \frac{B_{i,t}}{P_t}$ is household net worth and $v_t \in [0\ 1]$. I assume as in DG (2018): $Q_t F_{it} = \max[0, v_t A_{it}]$

- Intuition : Only household with positive net worth claim firm's profit (No short selling: $F_{it} \geq 0$).
- Under the Assumption : $F_{it} = \frac{A^+_{it}}{A^+_t}$ with $A^+_{it} = \max[0, A_{it}]$ and $A^+_t = \int_0^1 A^+_{it} di$.
- Only the asset A_{it} relevant.

Household problem

Transfer is assumed to follow the following rule:

$$T_{it} = \left[1 + \tau_t^a \left(\frac{A_{it}^+}{A_t^+} - 1\right) + \tau_t^e \left(e_{it} - 1\right)\right] \delta D_t$$

- Wealth-based rule (W-rule) : $\tau_t^a = 1$ and $\tau_t^e = 0$; all profit goes to asset holder. another transfer rule

Wage setting problem

Following Hagedorn et al. (2019), I assume there exist a middleman choses $\hat{W}_t = W_{it}$ by solving:

$$\max_{\hat{W}_t} \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(\hat{W}_t, \hat{W}_{t-1}, Z_t) di - \int_0^1 \frac{g\left(\hat{N}_t(\hat{W}_t, W_t, Z_t)\right)}{u'(C_t)} di \right] \quad (1)$$

$$s.t \quad \hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[\frac{W_t}{\hat{W}_t}\right]^{\epsilon_w} N_t \quad (2)$$

- g(.) is labor disutility.
- In red , aggregate benefice of labor per aggregate marginal utility.
- In blue , aggregate cost of labor per aggregate marginal utility.
- 2 is optimal labor demand from Employment agencies problem. details

Equilibrium conditions

$$C_{it}^{-\sigma} \geq \beta(1+r_t)\mathbb{E}(C_{it+1}^{-\sigma}) \tag{3}$$

$$\frac{\theta_p}{\varepsilon}\Pi_t^p\left(\Pi_t^p - 1\right) = \left[m_t - \frac{\varepsilon - 1}{\varepsilon}\right] + \frac{\theta_p}{\varepsilon}\beta\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_{t+1}^p\left(\Pi_{t+1}^p - 1\right)\frac{Y_{t+1}}{Y_t}\right] \tag{4}$$

$$\theta_w\Pi_t^w(\Pi_t^w - 1) = w_t(1-\epsilon_w) + \epsilon_w N_t^\eta C_t^\sigma + \beta\theta_w\mathbb{E}_t\left[\Pi_{t+1}^w(\Pi_{t+1}^w - 1)\frac{\mathbb{Z}_{t+1}}{\mathbb{Z}_t}\right] \tag{5}$$

- 3 is standard Euler Equation which hold with equality for unconstrained agents; 4 is NK Price Phillips Curve (from standard firm problem) and 5 is NK Wage Phillips Curve.
- Three equations + Taylor rule + exogenous shock process of the model summary the economy.
- Why can't solve this analytically: No possible aggregation for 3 but possible in TANK

TANK framework

- Time-invariant unconstrained agents of measures $1 - \lambda$ and time-invariant constrained agents of measures λ.
- $e_{it} = 1$ for every i at every t.
- $F_{it} = \frac{1}{1-\lambda}$ only for unconstrained agents.
- The standard Euler Equation holds with equality for unconstrained agents.
- Constrained agents doesn't participate on the bond market.
- At the equilibrium. $\frac{B_{i,t}}{P_t} = 0$

TANK: Characterization of the equilibrium

Proposition 1: Under sticky wages and sticky prices the following system of 4 equations summarizes the equilibrium.

$$\begin{cases} \pi_t^p = \beta \mathbb{E} \pi_{t+1}^p + \lambda_p \tilde{w}_t + k_p \tilde{y}_t & \text{Price NKPC} \\ \pi_t^w = \beta \mathbb{E} \pi_{t+1}^w - \lambda_w \tilde{w}_t + k_w \tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E} \tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi_3)} \hat{r}_t^b + \frac{\Psi_2}{1+\Psi_3} \mathbb{E} \left[\tilde{w}_{t+1} - \tilde{w}_t \right] & \text{DIS} \\ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \end{cases} \tag{6}$$

- There are two differences with the standard “3 equations” model:
  - there is one Phillips curve for each source of nominal rigidity
  - The aggregation only alters the demand side of the model: the Dynamic IS equation and the change is proportional to the change in the wage gap.

TANK: Aggregation 1/3

Lemma 1: Two measures are necessary and sufficient to aggregate the consumption in the economy. The two measures are:

- The consumption of unconstrained agents.
- A measure of consumption inequality.

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Proof. By definition, the aggregate consumption is: $C_t = (1-\lambda)C_t^U + \lambda C_t^K$ or equivalently $C_t = C_t^U(1-\lambda\gamma_t)$, where $\gamma_t = \frac{C_t^U - C_t^K}{C_t^U}$. Linearize around the steady state gives: $\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\hat{\gamma}_t$

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The dynamics $\hat{c}_t^U$ is known using the Euler equation for unconstrained agents. Next lemma characterizes the change in consumption inequality $\hat{\gamma}_t$.

TANK: Aggregation 2/3

Lemma 2: The change in consumption inequality around the steady state is proportional to the percentage change in real price markup around the steady state.

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Proof.

- At the equilibrium, $C_t^U - C_t^K = D_t \left(\frac{1-(1-\tau)\delta}{1-\lambda} \right)$, Where D_t is firm's profit.
- The profit $D_t = Y_t - w_t N_t - AC_t = \left[\left(1 - \tilde{AC}_t \right) - (1-\alpha)\, m_t \right] Y_t$ where $m_t = \frac{w_t}{MPN}$ is the inverse of the real price markup.
- The markup determines the profit which determine the consumption inequality.
- Up to first order approximation, $\hat{\gamma}_t = -\Psi_1 \hat{\mu}_t^p$, where $\Psi_1 < 0$ and $\hat{\mu}_t^p$ is the real price markup deviation from its steady value $\mu^p = \frac{\epsilon_p}{\epsilon_p - 1}$
- In addition using the definition of the price markup: $\hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha} \tilde{y}_t$

TANK: Aggregation 3/3

- Under lemma 1 and lemma 2, the aggregate cons. is $\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\Psi_1\left[\tilde{w}_t + \frac{\alpha}{1-\alpha}\tilde{y}_t\right]$

- Using the Euler equation for unconstrained agents, we have $\hat{c}_t^U = \mathbb{E}\hat{c}_{t+1}^U - \frac{1}{\sigma}\hat{r}_t^b$

- Using the aggregate resource constraint, up to first-order approximation the percentage change in aggregate consumption around the steady state is equal the percentage change in output around the steady state. That is $\hat{c}_t = \hat{y}_t$

- By definition, $\hat{y}_t = \tilde{y}_t + \hat{y}_t^n$, where $\hat{y}_t^n = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)}a_t$. For a monetary policy shock $\hat{y}_t^n = 0$

- By combining the above points, we obtain the Dynamic IS equation shown in proposition 1.

TANK: sticky wages

Proposition 2 : Under sticky wages (and flexible prices), the DIS equation $\tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi_3)}\hat{r}_t^b + \frac{\Psi_2}{1+\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right]$ is reduced to $\tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma}\hat{r}_t^b$.

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- Hence TANK is equivalent to RANK
- Intuition: the change in consumption inequality is the key driving the difference between RANK and TANK.
- Only the percentage change in real price markup determines the change in consumption inequality.
- under flexible prices,the percentage change in real price markup is 0 (lemma 1)

IRF RANK TANK

What do we know at this stage

- Under sticky wages and flexible prices, TANK is equivalent to RANK.
- Have the equations that summarize the HANK equilibrium: but can not solve it analytically.
- Next step: Use numerical method to solve for the HANK equilibrium.
  - Compare HANK VS TANK under sticky prices and sticky wages .

HANK: Numerical solution

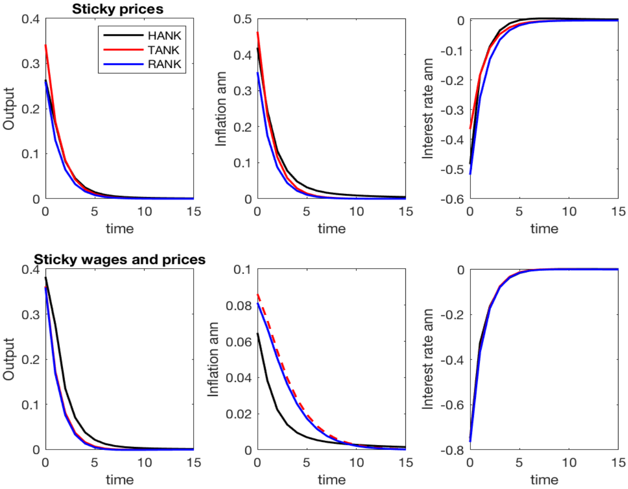
- Step 1: Solve for stationary distribution Stat distribution
  - I use Endogenous Grid point Method (EGM).
  - Debortoli and Gali (2018) use Reiter (2010) method.

- Step 2: Solve for the aggregate fluctuation
  - Schmitt-Grohé and Uribe (2004); Bayer et al. (2019)
  - Solve a full system of 1769 equations of form:

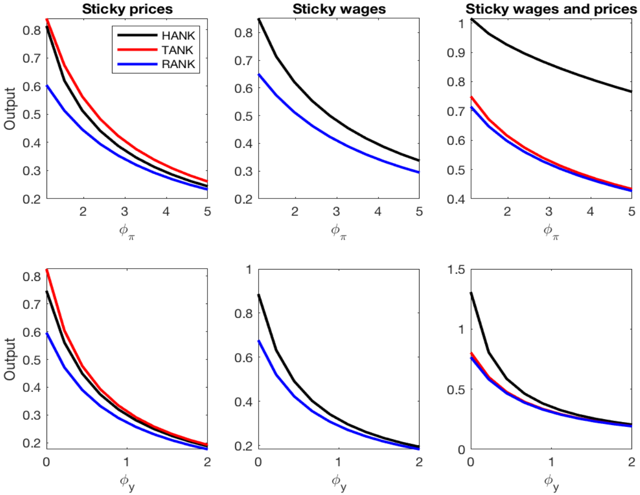
$$\mathbb{E}\left[X_t, X_{t+1}, Y_t, Y_{t+1}\right] = 0.$$

Calibration

Aggregate fluctuations: IRF of MP shock



Aggregate fluctuations: Cumulative response, MP shock



Conclusion

- Under sticky prices and sticky wages, a Two-Agents New Keynesian (TANK) model cannot approximate Heterogeneous-Agents New Keynesian (HANK) model.
- The presence of sticky wages limits the role of Hand to Mouth in TANK.

Transfer rule

Transfer is assumed to follow the following rule:

$$T_{it} = \left[1 + \tau_t^a \left(\frac{A_{it}^+}{A_t^+} - 1 \right) + \tau_t^e \left(e_{it} - 1 \right) \right] \delta D_t$$

- Wealth-based rule (W-rule) : $\tau_t^a = 1$ and $\tau_t^e = 0$; all profit goes to asset holder.
- Productivity-based rule (P-rule) : $\tau_t^a = 0$ and $\tau_t^e = 1$; profit is shared proportional to household labor income risk.
- Uniform (U-rule) : $\tau_t^a = 0$ and $\tau_t^e = 0$; lump sum transfer same for every household.

back to transfer rule

Employment agencies problem

Following Erceg et al. (1999)

$$N_t = \left[\int_0^1 e_{it} \left(N_{it} \right)^{1-\frac{1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}$$

Where ϵ_W is the elasticity of substitution across labor services

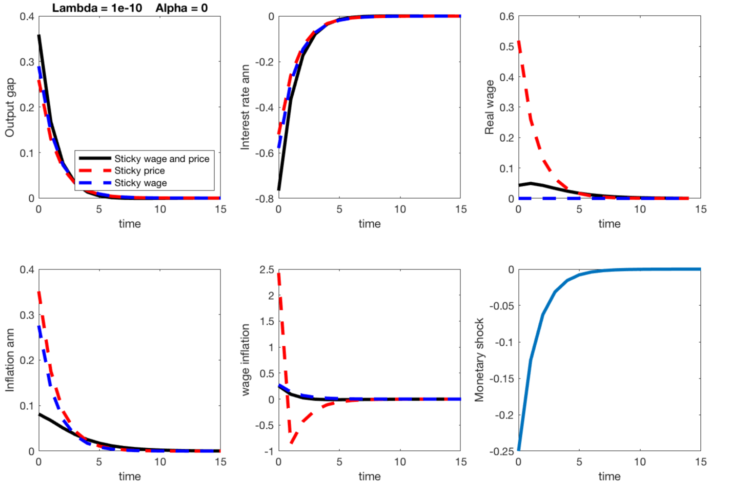
$$\begin{aligned} \max_{N_{it}} W_t N_t \quad &- \quad \int_0^1 W_{it} N_{it} e_{it} \\ s.t \quad N_t \quad &= \quad \left[\int_0^1 e_{it} \left(N_{it} \right)^{1-\frac{1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}} \end{aligned}$$

The solution (the demand for the i-th consumer's labor) is:

$$N_{it} = \left[\frac{W_t}{W_{it}} \right]^{\epsilon_W} N_t$$

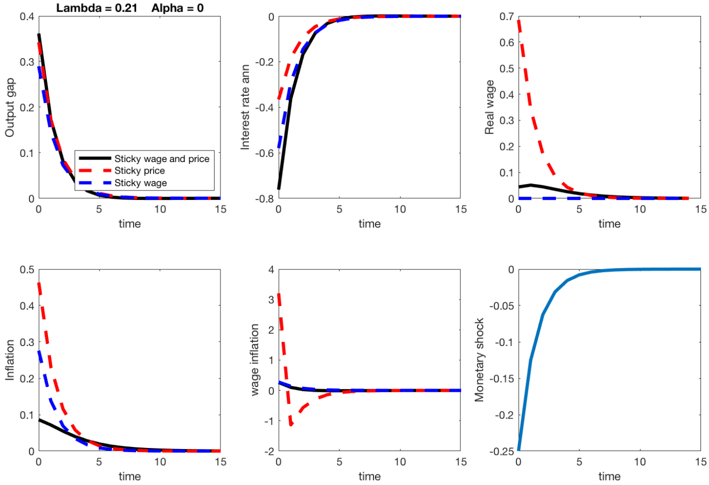
back to wage setting

MP shock: RANK



back to Prop1

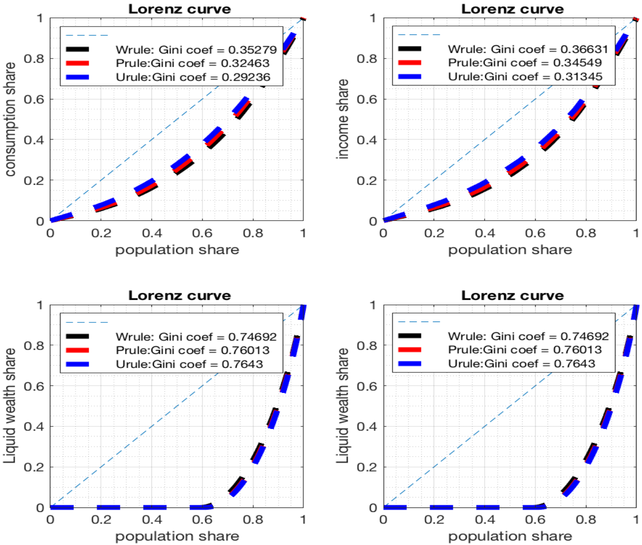
MP shock: TANK



back to Prop1

Solution for stationary distribution

back to step



Calibration

	Parameter	Description	Target/source
Debortoli and Gali (2018)	$\beta = \begin{cases} 0.9745 & W-rule \\ 0.9743 & P-rule \\ 0.9679 & U-rule \end{cases}$	Disc factor	avg real interest $\bar{r} = 3\%$
	$\Psi = 0.5$	Borr limit	share of constr. 21% -27%

	Parameter	Description	Target/source
My calibration	$\beta = \begin{cases} 0.9778 & W-rule \\ 0.9773 & P-rule \\ 0.9799 & U-rule \end{cases}$	Disc factor	avg real interest $\bar{r} = 3\%$
	$\Psi = 0.5$	Borr limit	share of constr. 21.7% -26.8%

back to step