Monetary Policy, Sticky Wages, and Household Heterogeneity [1]

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Abstract

Can a Two-Agent New Keynesian model, also known as TANK, approximate a Heterogeneous Agents New Keynesian (HANK) model in terms of its aggregate response to a monetary policy shock? In this paper, I show that the answer depends on the source of nominal rigidities. If prices are sticky, the answer is yes, as shown by Debortoli and Gali (2018). If wages are sticky, the answer is no, as shown in this paper. To make this point, I show that the TANK model with only wage rigidities is equivalent, in terms of aggregate variables, to the representative agent New Keynesian model. For TANK with both price and wage rigidities, I show numerically that TANK does not approximate HANK well.

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1 Introduction

What is the transmission mechanism of a monetary policy shock in an economy? What are the responses of aggregate variables, such as GDP and consumption, to a monetary policy shock? Those are some classic questions addressed in the large monetary economics literature within a Representative Agent New Keynesian (i.e., RANK) model. In recent years, Heterogeneous Agents New Keynesian models often referred to as HANK models, have gained attention and have substantially revised our understanding of the answers to these questions[3]. However, the HANK model features a continuous joint distribution of income and wealth and lacks tractability. In this paper, I study the extent to which a Two-Agent New Keynesian (TANK) model with limited heterogeneity can approximate a HANK model in terms of the response of aggregate variables, such as GDP and consumption, to a monetary policy shock. I argue that whether TANK models provide a good approximation to HANK models depends on the considered nominal rigidities.

Understanding the condition under which the TANK model approximates the HANK model is important for the following reasons. First, the HANK model does not have a closed-form solution since it requires to keep track of the wealth distribution as a state variable. We can only rely on a nontrivial numerical solution to solve for the equilibrium of HANK economies. This lack of analytical tractability poses challenges for the identification of the economic mechanisms underlying the results[4]. Second, nominal rigidities is the source of monetary non-neutrality in these models. Rigidities in the price of consumption and wages have been documented (Taylor (1999)). Thus, it is important to know if the approximation of HANK by TANK in terms of aggregate fluctuations depends on the considered nominal rigidity.

To answer this question, I use a general equilibrium framework as in Debortoli and Galí (2018) in which I introduce sticky wages and a monopolistically competitive labor market. I introduce sticky wages for three reasons. First, the New Keynesian literature has empirically documented that wages

[3] In seminal work, Kaplan et al. (2018) studied the transmission of monetary policy in HANK model to household consumption. In contrast to the RANK model, they found that in HANK, monetary policy works mostly through the income effect as opposed to the inter-temporal substitution effect. The substitution effect captures the extent to which households save less (or borrow more) to increase consumption when the real interest rate declines. The income effect captures the general equilibrium effect where the decline in the real interest rate affects labor demand and thus labor income.

[4] Acharya and Dogra (2020) study a full tractable HANK model with CARA utility function.

are as sticky as prices (Taylor (1999)). Second, besides their empirical relevance, wage rigidities have been shown to be qualitatively and quantitatively important for the modeling of economies with a role for monetary policy. When wages are rigid, output exhibits persistence in its response to a monetary shock, which is in line with the response observed in the data (Christiano et al. (2005)). For models with heterogenous agents where a fraction of households live as "hand-to-mouth", wage rigidities play an important role in keeping the volatility of real income in line with the one observed in the data. Third, sticky wages have been shown to preserve the "standard aggregate demand logic" and the relevance of the Taylor principle for a plausible calibration of the share of hand-to-mouth households (Bilbiie (2008), Colciago (2011), Ascari et al. (2011)) [5].

In this paper, I first show that under sticky wages (and flexible prices), a TANK model is equivalent to a RANK model in terms of its response to demand and supply shocks. The intuition behind this result is as follows. It is worth noting that the differences between RANK and TANK models are twofold. First, in the TANK model, there is a fixed fraction of households that cannot borrow. Second, profit is not redistributed uniformly because profit is generally shared between asset holders. At the equilibrium, there is no trade in bond in the equilibrium of the RANK and TANK models as asset holders are identical. Therefore, the only source of difference between RANK and TANK is how the firms' profit are distributed. If a firm's profit is uniformly redistributed to all households in the TANK model, then TANK will be equivalent to RANK whatever the type of nominal rigidity is in the economy.

In monopolistic competition, the firm's profit is proportional to the price markup and output. Given that the profit rate (profit over output) depends only on the markup, the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market), is proportional to the price markup. If prices are flexible (and wages are sticky), the firm can always adjust its price for a constant price markup. Every firm faces the same nominal wage set by the wage union. So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK. So why is HANK not equivalent to RANK when prices are flexible? The reason is simple. In HANK, the consumption inequality

[5] For a plausible proportion of Hand-to-Mouth, TANK models lead to a situation where the Taylor principle is no longer a necessary condition for equilibrium determinacy, and the Standard Aggregate Demand Logic (an increase in real interest rate leads to a decrease in aggregate consumption) does not hold.

does not only come from the ownership of the firms but also from the idiosyncratic labor income risk and debt choice. To summarize, TANK is equivalent to RANK under sticky wages and flexible prices because the consumption gap between constrained and unconstrained agents is constant over time so there is no change in the consumption inequality.

With inequality, not only the type of nominal rigidity matters but also the source of nominal rigidity because redistribution matters (see Auclert (2017) who shed light on the role of redistribution in the transmission of the monetary policy). Wage markup uniformly impacts every household in my work because wages are set outside the firms. On the contrary, price markup is not distributed uniformly. In general, only asset holders gain from a price markup since they own the firms.

Second, I find quantitatively as Debortoli and Galí (2018) that under sticky prices, TANK approximates HANK well in terms of its response to an aggregate shock. Unlike Debortoli and Galí (2018), under sticky prices and sticky wages, I find that TANK can no longer approximate HANK. Building on the intuition above, sticky wages mute the response of the gap between the consumption of hand to mouth and other households in response to an aggregate shock.

The extent to which the heterogeneity[6] affects aggregate fluctuations has been studied by many authors including (Werning (2015); Acharya and Dogra (2020); Bilbiie (2019)). Debortoli and Galí (2018) also offer a better understanding of how the heterogeneity affects aggregate fluctuations in response to a demand shock (monetary policy shock and preference shock) and supply shock (technology shock). Based on the structure of constrained and unconstrained agents in HANK, Debortoli and Galí (2018) upon a first order linear approximation show analytically that the HANK framework is different from the RANK framework along three dimensions: the change in the consumption gap between constrained and unconstrained agents, the change in the consumption dispersion within unconstrained agents and the change in the share of constrained agents. It is not possible to analytically compute the three statistics since it requires to know the wealth distribution at each point in time. For this reason, Debortoli and Galí (2018) build on Bilbiie (2008) a Two Agents New Keynesian model often referred to as TANK. Three statistics summarize the state of the economy in the TANK model. The advantage of the TANK framework is that the three statistics (the change in the consumption gap

[6]In most of the HANK literature, the assumed heterogeneity is often of the form postulated by Aiyagari (1994), where each household faces uninsurable idiosyncratic labor income risk and a borrowing limit.

between constrained and unconstrained agents, the change in the consumption dispersion within unconstrained agents and the change in the share of constrained agents) can be computed analytically. Note that, by construction, all three are zero in RANK while TANK allows to focus on the consumption gap between constrained and unconstrained agents. Under sticky prices Debortoli and Galí (2018) find that TANK approximates well HANK in terms of aggregate fluctuations both for demand and supply shock.

My paper mainly relates to the recent literature on HANK Werning (2015); Gornemann et al. (2016); Bilbiie (2019); Auclert (2017); Kaplan et al. (2018); Luetticke (2018); Bayer et al. (2019); Acharya and Dogra (2020). I contribute to this literature by studying sticky wages. To my knowledge, Hagedorn et al. (2019b), Hagedorn et al. (2019a) are the first papers introducing sticky wages in a general HANK framework. While they focus on the fiscal multiplier and forward guidance, I offer detailed comparison between a non-tractable HANK model and a tractable TANK model. My work is closely related to Debortoli and Galí (2018) who are the first to offer a better understanding of the difference between HANK and TANK in terms of its response to aggregate shocks. I introduce sticky wages, which play a key role in the comparison between TANK and HANK.

My work is also related to earlier literature on two agents model as Bilbiie (2008); Colciago (2011); Ascari et al. (2011). While they focus on the comparison between RANK and TANK, I build on their work by comparing a Two-Agent New Keynesian model to a Heterogeneous Agents New Keynesian model.

The rest of the paper is organized as follows. In Section 2 I present the HANK model. Section 3 presents the TANK framework. Section 4 presents my finding and discusses some findings in Debortoli and Galí (2018) and Section 5 concludes.

2 Model

I build a dynamic stochastic model with household heterogeneity. The household faces labor income risk and a borrowing limit à la Aiyagari (1994). There is a monopolistic competitive firm that faces sticky prices. Wages are sticky in the spirit of Erceg et al. (2000).

2.1 Household

There is a continuum of ex-ante identical households of measure one indexed by their liquid asset B, their share of the equity fund F and their uninsurable labor income risk e. Labor income risk follows a markov process. Households self-insure against the labor income risk by saving in the liquid asset B. By purpose, the household side is kept as close as possible to Debortoli and Galí (2018). A household i choses C_{it} to maximize his expected discounted utility $\mathbb{E}\sum_{t=0}^{\infty}\beta^t U(C_{it}, N_{it})$, where $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$, subject to its current budget constraint:

$$C_{it} + Q_t F_{it} + \frac{B_{i,t}}{P_t} = \frac{B_{it-1}(1+i_{t-1})}{P_t} + w_t N_t e_{it} + \left[Q_t + (1-\delta)D_t\right] F_{it-1} + T_{it} - e_{it}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}_t,$$

where $\mathbb{Z}_t$ is an aggregate variable (aggregate output Y_t for instance) taken as given by households. The budget constraint is as in Debortoli and Galí (2018) except for the wage adjustment cost. The right hand side of the budget constraint is composed of bond income, labor income, equity income, transfer income and the wage adjustment cost. A share $1-\delta$ of firm's profit D_t is claimed by equity fund holders. The remaining share δ of firm's profit is transferred to household from a specific rule described below. The real wage is w_t and Q_t is the price of the equity fund. Each household is subject to a borrowing limit of the form:

$$\frac{B_{i,t}}{P_t} \quad \geq \quad -\Psi Y,$$

where Y is the yearly output. The equity share F_{it} is assumed to be non negative; that is there is no short selling. As Debortoli and Galí (2018), we assume that: $Q_t F_{it} = \max[0, v_t A_{it}]$, where A_{it} is the net worth given by: $A_{it} = Q_t F_{it} + \frac{B_{it}}{P_t}$ and $v_t \in [0\ 1]$. With $\int_0^1 F_{it} di = 1$, one can show that $F_{it} = \frac{A_{it}^+}{A_t^+}$ where $A_{it}^+ = \max[0, A_{it}]$ and $A_t^+ = \int_0^1 A_{it}^+ di$.

The transfer is assumed to follow the below rule:

$$T_{it} = \left[1 + \tau_t^a\left(\frac{A_{it}^+}{A_t^+} - 1\right) + \tau_t^e\left(e_{it} - 1\right)\right]\delta D_t.$$

From this transfer rule, three cases are considered: the first one is the Wealth-based rule $(W-rule)$

where $\tau_t^a = 1$ and $\tau_t^e = 0$; the second one is the productivity-based rule $(P - rule)$ where $\tau_t^a = 0$ and $\tau_t^e = 1$; and the third one is the Uniform-based rule $(U - rule)$ where $\tau_t^a = 0$ and $\tau_t^e = 0$. In the $W - rule$ the illiquid profit δD_t is distributed only to current share holders. The $P - rule$ shares the illiquid profits among households proportionally to their labor productivity. In the $U - rule$, the illiquid profit is equally shared between all households (See Debortoli and Galí (2018) for more details).

Let's $b_{it} = \frac{B_{it}}{P_t}$, $1 + r_t = \frac{1+i_t}{\Pi_{t+1}}$, and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. The household's problem gives the standard Euler equation with inequality:

$$U_c(C_{it}, N_t) \geq \beta(1 + r_t)\mathbb{E}(U_c(C_{it+1}, N_{t+1}). \tag{1}$$

This standard Euler equation holds with equality for unconstrained agent (household for whom the credit limit is not binding). One additional unit of consumption today increases its utility by $U_c(c_t)$. If the household saves this unit of consumption in a the riskless bond, it gains tomorrow $(1+r_t)U_c(c_{t+1})$, where r_t is the riskless real interest rate. At the optimum the cost of saving should be equal to its discounted benefit for unconstrained agents.

Following Debortoli and Galí (2018), we assume that the equity share price Q_t is the discounted expected of all futures return of the equity share:

$$Q_t = \mathbb{E}_t(\Lambda_t^Q [Q_{t+1} + (1 - \delta)D_{t+1}]), \tag{2}$$

where Λ_t^Q is the stochastic discounted factor. The relevant stochastic discounted factor is given by: $\Lambda_t^Q = \beta \frac{U_c(C^+_{t+1/t})}{U_c(C_t^+)}$ where C_t^+ and $C^+_{t+1/t}$ are consumption in period t and t+1 of households with positive net wealth in period t, weighted by their share holding or their wealth.

2.2 Employment Agencies

We assume as in Erceg et al. (2000) that there exists a perfectly-competitive employment agencies which hire the differentiated labor of consumers and aggregate them using the CES technology.

$$N_t = \left[\int_0^1 e_{it} \left(N_{it}\right)^{1-\frac{1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{3}$$

Where ϵ_w is the elasticity of substitution across labor services and e_{it} is the uninsurable idiosyncratic labor income risk. The profit function of the agency is given by: $W_t N_t - \int_0^1 W_{it} N_{it} e_{it}$. The solution to the profit maximization (in appendix A.1) gives the labor demand:

$$N_{it} = \left[\frac{W_t}{W_{it}}\right]^{\epsilon_w} N_t. \tag{4}$$

Equation 4 states that with the same nominal wage among household that is $W_{it} = W_{jt}$ for every i, j then the labor supply is the same across household. This implies that the observed difference of labor income across household comes from the idiosyncratic risk. The volatility of this labor income crucially depends on the variance of the idiosyncratic risk.

Competition implies that profits are null in equilibrium. We obtain the wage index W_t by replacing the solution N_{it} into the profit function, which yields the following wage index:

$$W_t = \left[\int_0^1 (W_{it})^{1-\epsilon_w}\, di\right]^{\frac{1}{1-\epsilon_w}}. \tag{5}$$

2.3 Wage setting

Following Hagedorn et al. (2019b), we assume that there is a middleman who sets the nominal wage $\hat{W}_t$ for an effective unit of labor such that $\hat{W}_t = W_{it}$ and $N_{it} = \hat{N}_t$. Since wages are sticky, a change in current wages with respect to past wages is subject to an adjustment cost à la Rotemberg (1982). This adjustment cost for a household is proportional to the realization of the idiosyncratic risk and is given by: $\Theta_{it}(W_{it}, W_{it-1}, Z_t) = e_{it}\frac{\theta_w}{2}\left(\frac{W_{it}}{W_{it-1}} - 1\right)^2 Z_t$ where Z_t is aggregate output.

For the middleman the benefit for an effective labor is given by: $\int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(\hat{W}_t, \hat{W}_{t-1}, Z_t) di$ with $\hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[\frac{W_t}{\hat{W}_t}\right]^{\epsilon_w} N_t$. The cost is given by $\int_0^1 \frac{g\left(\hat{N}_t(\hat{W}_t, W_t, Z_t)\right)}{u'(C_t)} di$, where $g(N_t) = \frac{N_t^{1+\eta}}{1+\eta}$ is labor dis-utility and $u'(C_t)$ is the aggregate marginal utility which is present in the cost because we abstract it from the benefit function.

The middleman solves the following problem:

$$\max_{\hat{W}_t} \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(\hat{W}_t, \hat{W}_{t-1}, Z_t) di - \int_0^1 \frac{g\left(\hat{N}_t(\hat{W}_t, W_t, Z_t)\right)}{u'(C_t)} di \right] \tag{6}$$

$$s.t \quad \hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[\frac{W_t}{\hat{W}_t}\right]^{\epsilon_w} N_t.$$

Solving the problem in 6 using $\hat{W}_t = W_t$ yields the wage Phillips curve:

$$\theta_w \Pi_t^w (\Pi_t^w - \bar{\Pi}^w) = w_t(1 - \epsilon_w) + \epsilon_w N_t^\eta C_t^\sigma + \beta \theta_w \Pi_{t+1}^w (\Pi_{t+1}^w - \bar{\Pi}^w) \frac{\mathbb{Z}_{t+1}}{\mathbb{Z}_t}, \tag{7}$$

where $\Pi_t^w = \frac{W_t}{W_{t-1}}$ is the nominal wage inflation, and $w_t = \frac{W_t}{P_t}$ is the real wage. Let denote μ_t the real wage markup. We have : $\frac{W_t}{P_t} = \mu_t MRS_t$, where $MRS_t = -\frac{U_N}{U_C}$ is the marginal rate of substitution.

Condition 7 is the non linear version of New-Keynesian Wage Phillips Curve. Note that when wage is fully flexible (ie $\theta_w = 0$), $w_t = \frac{\varepsilon_w}{\varepsilon_w - 1} N_t^\eta C_t^\sigma$ that is the real wage is equal to the product of the labor wedge and the marginal rate of substitution between consumption and labor. The labor wedge $\frac{\varepsilon_w}{\varepsilon_w - 1}$ is equivalent to the steady state real wage markup raised in this set up because of monopolistic labor market. When wage is fully rigid (ie $\theta_w \longrightarrow \infty$) $\Pi_t^w = 1$, for all t. The middleman sets wage once for all and never updates it.

Linearize 7 around the deterministic steady state yields:

$$\pi_t^w = \beta \mathbb{E} \pi_{t+1}^w - \frac{\epsilon_w w}{\theta_w} \hat{\mu}_t^w. \tag{8}$$

Condition 8 is the linear version of New-Keynesian Wage Phillips Curve. The log deviation of the firm's real markup from its steady state is denoted by $\hat{\mu}_t^w$. It states that if household real wage markup is below their natural level (equivalent to the steady-state level), the middleman resets nominal wage up which increases wage inflation. When wages are fully rigid (ie $\theta_w \longrightarrow \infty$) $\pi_t^w = 0$, for all t and the wage inflation will be 0 for any shock.

2.4 Firms

There are monopolistically competitive intermediate good producing firms and perfectly final goods producing firms that aggregate differentiated intermediate goods into a single good Y_t. Using the Rotemberg (1982) adjustment cost $AC_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \pi\right)^2 Y_t$, the solution to the firm's problem gives the following condition:

$$\pi_t^p = \beta \mathbb{E} \pi_{t+1}^p - \frac{\varepsilon - 1}{\theta} \hat{\mu}_t^p. \tag{9}$$

Condition 9 is the linear version of the New-Keynesian Price Phillips Curve. The elasticity of substitution across good is denoted by ϵ, π_t^p is the price inflation, and $\hat{\mu}_t$ is the log deviation of firm real markup from his steady state. It states that if firms' markup are below their natural level (equivalent to the steady state level), it resets prices up, which increases inflation.

2.5 Monetary authority

The monetary authority uses taylor-rule to set nominal interest as follows:.

$$\hat{i_t} = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t, \tag{10}$$

where v_t represents exogenous monetary policy shocks and it follows an AR(1) process.

3 Two Agents: TANK framework

We assume two types of agents. The first type is the time-invariant unconstrained agents U of measures $1-\lambda$, and the second type is the time-invariant constrained agents K of measures λ. The time-invariant unconstrained agents U are not constrained on bond market and the time-invariant constrained agents K do not participate to the bond market. The agents U own firms and claim the a part of aggregate profit $(1-\delta)D_t$ as in Debortoli and Gali (2018). There is no idiosyncratic risk in TANK: $e_{it} = 1$.
The household budget constraint for constrained agents U is given by:

$$C_{1t} + \frac{1}{1+r_t} b_{1,t+1} = b_{1t} + \frac{1}{P_t} W_{1t} N_{1t} + \left[Q_t + (1-\delta)D_t\right] F_{it-1} + T_{1t} - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_{1t}}{W_{1t-1}} - 1\right)^2 \mathbb{Z}_t. \tag{11}$$

The household budget constraint for constrained agents K is given by:

$$C_{2t} = \frac{1}{P_t} W_{2t} N_{2t} + T_{1t} - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_{1t}}{W_{2t-1}} - 1\right)^2 \mathbb{Z}_t. \tag{12}$$

Since there is no heterogeneity between unconstrained agents, at the equilibrium: $F_{it-1} = \frac{1}{1-\lambda}$ and $b_{1,t+1} = 0$ the budget constraints of households are given by: $C_t^U = w_t N_t + \frac{1-\delta}{1-\lambda} D_t + T_t^U - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}$ and $C_t^K = w_t N_t + T_t^K - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}_{\approx}$.Where w_t is the real wage. Following Debortoli, Gali (2018), $T_t^U = \left(1 + \frac{\tau\lambda}{1-\lambda}\right)\delta D_t$ and $T_t^K = (1-\tau)\,\delta D_t$. Note that $(1-\lambda)T_t^U + \lambda T_t^K = \delta D_t$ For $\tau = 1$, all the profits end up in the hands of unconstrained agents (W-rule), for $\delta = 1$ and $\tau = 0$, all profits are sharing equally between unconstrained and constrained households(U-rule and P-rule).

The Euler equation of unconstrained agents is given by:$Z_t C_t^{U-\sigma} = \beta(1+r_t)\mathbb{E}\left[Z_{t+1} C_{t+1}^{U\ -\sigma}\right]$. The linearization of this Euler equation yields:

$$\hat{c}_t^U = \mathbb{E}\hat{c}_{t+1}^U - \frac{1}{\sigma}\hat{r}_t - \frac{1}{\sigma}\mathbb{E}\Delta z_{t+1} \tag{13}$$

Equation 13 depends on the percentage change of unconstrained agents' consumption. The goal is to have a version of equation 13 that depends only on aggregate variables. The new equation will be the Dynamics Investment-Saving (DIS) curve. In HANK equilibrium, it was impossible to aggregate the economy because of the uninsurable idiosyncratic labor risk. The TANK framework allows us to aggregate because there is no endogenous transition between unconstrained and constrained states. Indeed the state in which the agents belong is permanent and exogenous. The following three lemma help us to aggregate the economy.

Lemma 1: Two measures are necessary and sufficient to aggregate the consumption in the economy. The two measures are: the consumption of unconstrained agents and a measure of consumption inequality. That is:

$$\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\hat{\gamma}_t \tag{14}$$

Proof. By definition, the aggregate consumption is: $C_t = (1-\lambda)C_t^U + \lambda C_t^K$ or equivalently $C_t = C_t^U(1-\lambda\gamma_t)$, where $\gamma_t = \frac{C_t^U - C_t^K}{C_t^U}$. Linearize around the steady state gives: $\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\hat{\gamma}_t$. The dynamics of $\hat{c}_t^U$ expressed in equation 14 is known using the Euler equation for unconstrained agents. Equation 14 tells us two things. First, the dynamics of the aggregate consumption depends on two factors: the dynamics of the consumption of unconstrained agents and the dynamics of the consumption inequality (the consumption inequality is the ratio of the consumption of constrained agents to the consumption of unconstrained agents). Second, if the consumption inequality is constant over time, then the dynamic of the aggregate consumption is the same as the dynamics of the consumption of unconstrained agents.We need now to find an analytical expression for $\hat{\gamma}_t$. Next lemma describes the change in consumption inequality $\hat{\gamma}_t$.

Lemma 2: The change in consumption inequality around the steady state is proportional to the percentage change in real price markup around the steady state. That is:

$$\hat{\gamma}_t = \Psi_1 \hat{\mu}_t^p, \tag{15}$$

where $\Psi_1 = -\gamma_m m$; $\Psi_1 > 0$ and $\hat{\gamma}_t = \gamma_t - \gamma$

Proof (see the complete proof in appendix B.2). At the equilibrium, $C_t^U - C_t^K = D_t\left(\frac{1-(1-\tau)\delta}{1-\lambda}\right)$, Where D_t is firm's profit. The profit $D_t = Y_t - w_t N_t - AC_t = \left[\left(1 - \tilde{AC}_t\right) - (1-\alpha)\, m_t\right] Y_t$, where $m_t = \frac{w_t}{MPN}$ is the inverse of the real price markup. The markup determines the profit which affects the consumption inequality. Up to first order approximation, $\hat{\gamma}_t = \Psi_1 \hat{\mu}^p_t$, where $\Psi_1 > 0$ and $\hat{\mu}^p_t$ is the real price markup deviation from its steady value $\mu^p = \frac{\epsilon_p}{\epsilon_p - 1}$. In addition, using the definition of the price markup: $\hat{\mu}^p_t = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t$

Equation 15 states that when the real markup increases the consumption gap between unconstrained and constrained agents increases. As expected, higher price markup negatively affects the Hand-to-Mouth's consumption.

Lemma 3: The percentage change in real price is proportional to the output gap and the real wage gap.

$$\hat{\mu}^p_t = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t. \tag{16}$$

Proof: By definition, $M^p_t = \frac{w_t}{MPN_t}$ then: $log(M^p_t) = \log(w_t) - \log(MPN_t) = (w_t) - log(1-\alpha) + \frac{\alpha}{1-\alpha} logY_t - \frac{1}{1-\alpha} logA_t$. This implies that: $\hat{\mu}^p_t = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t$.

Equation 16 states that given output gap, the price markup has a negative correlation with the real wage gap. Higher real wage implies a higher marginal cost for firms which lowers the firm's markup. In sticky wage equilibrium, $\hat{\mu}^p_t = 0$, thus $\tilde{w}_t = -\frac{\alpha}{1-\alpha}\tilde{y}_t$. A higher real wage gap induces a decline in the output gap. This decline is through the decline in labor supply. The presence of hand-to-mouth does not affect the relationship between the real wage gap and the output gap in sticky wage equilibrium.

3.1 Derivation of IS curve

Now, with lemma1 to lemma3 in hands, we can characterize the Dynamics Investment-Saving curve in function of aggregate variables. Combining equations 14, 15, and 16 gives $\hat{c}^U_t = \hat{c}_t - \frac{\lambda}{1-\lambda\gamma}\Psi_1 \left(\tilde{w}_t + \frac{\alpha}{1-\alpha}\tilde{y}_t\right)$

$$\hat{c}^U_t = \hat{c}_t - \Psi_3 \tilde{y}_t - \Psi_2 \tilde{w}_t, \tag{17}$$

where $\Psi_3 = \frac{\lambda}{1-\lambda\gamma}\frac{\alpha}{1-\alpha}\Psi_1$ and $\Psi_2 = \Psi_1 \frac{\lambda}{1-\lambda\gamma}$. Using the fact that $\hat{c}^U_t = \mathbb{E}\hat{c}^U_{t+1} - \frac{1}{\sigma}\mathbb{E}\hat{r}^b_{t+1} - \frac{1}{\sigma}\mathbb{E}\Delta z_{t+1}$ and $\hat{y}_t = \hat{c}_t = \tilde{y}_t + \hat{y}^n_t$ where $\hat{y}^n_t = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)}a_t$. a_t denotes the technology shock.

$$\tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1-\Psi_3)}\left(\hat{r}^b_t - \hat{r}^n_t\right) - \frac{\Psi_2}{1-\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right], \tag{18}$$

where $\hat{r}^n_t = -\mathbb{E}\Delta z_{t+1} + \sigma \Psi_a \mathbb{E}\Delta a_{t+1}$ with $\Psi_a = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)}$. Note that $\mathbb{E}\Delta z_{t+1} = (\rho_z - 1)z_t$ and $\mathbb{E}\Delta a_{t+1} = (\rho_a - 1)a_t$. Condition 18 is the DIS equation. The aggregation alters the demand side of the model. First, the sensitivity of output gap to the real interest rate is now $\sigma(1-\Psi_3)$ instead of σ. For plausible parameters, $0 < (1-\Psi_3) < 1$ implies that the output gap tends to be more responsive to the change in real interest rates. The presence of Hand-to-mouth consumers amplifies the response of the output gap to the change in real interest rates because they react more to a change in their labor income which increases when the output increases. Indeed, the HtM consumers have a higher marginal propensity to consume. Second, the output gap is proportional to the change in the real wage gap.

3.2 Derivation of Wage and Price Phillips Curve

In this section, I characterize the wage and price Phillips curve.

Wage Phillips Curve From equation 8 I have: $\pi_t^w = \beta \mathbb{E}\pi_{t+1}^w - \frac{\epsilon_w w}{\theta_w}\hat{\mu}_t^w$, where $\hat{\mu}_t^w$ is the steady state log deviation of the real wage markup. I also have $w_t = M_t^w N_t^\eta C_t^\sigma$. By linearizing the equation, we get: $\tilde{w}_t = \hat{\mu}_t^w + \sigma \tilde{c}_t + \frac{\eta}{1-\alpha}\tilde{y}_t$. Since the gap in consumption is equal to the gap in output (that is $\tilde{c}_t = \tilde{y}_t$), we have; $\tilde{w}_t = \hat{\mu}_t^w + \left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t$. The real wage markup is given by:

$$\hat{\mu}_t^w = -\left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t + \tilde{w}_t. \tag{19}$$

If the real wage is above its natural level in only sticky prices equilibrium (that is $\hat{\mu}_t^w = 0$), $\tilde{w}_t = \left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t$. There is positive correlation between real wage gap and output gap. The presence of Hand-to-Mouth, does not turn off this correlation. The Wage Phillips curve is given by:

$$\pi_t^w = \beta \mathbb{E}\pi_{t+1}^w - \lambda_w \tilde{w}_t + k_w \tilde{y}_t, \tag{20}$$

Where $\lambda_w = \frac{\epsilon_w w}{\theta_w}$, $k_w = \lambda_w\left[\sigma + \frac{\eta}{1-\alpha}\right]$. When the real wage is about its natural level, the middleman resets the wage down. There is a negative relation between wage inflation and the real wage gap. But if the output is above its natural level, the middleman resets up the wage. So there is a positive relation between wage inflation and output gap.

Price Phillips Curve: Combining 9 and 23 gives: $\pi_t = \beta \mathbb{E}\pi_{t+1} + \frac{\varepsilon m}{\theta}\left(\tilde{w}_t + \frac{\alpha}{1-\alpha}\tilde{y}_t\right)$

$$\pi_t^p = \beta \mathbb{E}\pi_{t+1}^p + \lambda_p \tilde{w}_t + k_p \tilde{y}_t, \tag{21}$$

Where $\lambda_p = \frac{\varepsilon - 1}{\theta}$ and $Kp = \left(\frac{\alpha}{1-\alpha}\right)\lambda_p$. This is a version of price NKPC. Contrary to the wage Phillips curve, When the real wage is above its natural level, the firms adjust up their price because of a higher marginal cost. So there is a positive relationship between price inflation and the real wage gap. If the output is above its natural level, the firms adjust up their prices. So there is a positive co-movement between the inflation and the output.

Wage identity equation: In this framework of sticky wage and sticky prices we have an important identity equation that makes the link between wage inflation and price inflation. By definition, $\Delta\tilde{w}_t = \Delta w_t - \Delta w_t^n$.

Which implies that $\Delta\tilde{w}_t = (w_t - w_{t-1}) - (p_t - p_{t-1}) - \Delta w_t^n$. So the wage identity condition can be written as follows.

$$\Delta\tilde{w}_t = \pi_t^w - \pi_t^p - \Delta w_t^n. \tag{22}$$

Note that $\Delta w_t^n = 0$ in the case of a demand shock. For a supply shock, $\Delta w_t^n = \frac{1-\alpha\Psi_a}{1-\alpha}\Delta a_t$.

Brief detour: Neutrality of Monetary policy From 16 and 19 , I get:

$$\begin{cases} \hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t \\ \hat{\mu}_{1t}^w = -\left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t + \tilde{w}_t \end{cases} \tag{23}$$

It is clear from that system of equation that in both flexible wage and price equilibrium ($\hat{\mu}_t^p = 0$ and $\hat{\mu}_t^w = 0$) $\tilde{w}_t = 0$ and $\tilde{y}_t = 0$ which means that the output gap and the real wage gap are zero. Thus, in absence of nominal rigidity, a monetary policy shock does not have any real effect on real variables: the so- called the neutrality of monetary policy in the absence of nominal rigidity.

3.3 Characterization of the equilibrium in TANK

In this section, I characterize the equilibrium in the TANK framework. The first proposition summarizes the equations in TANK equilibrium. The second proposition shows one of our main results: the observational equivalence between RANK and TANK in a flexible price and sticky wage environment.

Proposition 1 *Under sticky wages and prices, the TANK model can be summarized with the following system of 5 equations*

$$\begin{cases} \pi_t^p = \beta\mathbb{E}\pi_{t+1}^p + \lambda_p\tilde{w}_t + k_p\tilde{y}_t & \text{Price NKPC} \\ \pi_t^w = \beta\mathbb{E}\pi_{t+1}^w - \lambda_w\tilde{w}_t + k_w\tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1-\Psi_3)}\left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma\Psi_a\mathbb{E}\Delta a_{t+1}\right] - \frac{\Psi_2}{1-\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi\pi_t + \phi_y\hat{y}_t + v_t & \text{Taylor rule} \\ \Delta\tilde{w}_t = \pi_t^w - \pi_t^p - \frac{1-\alpha\Psi_a}{1-\alpha}\Delta a_t & \text{Identity} \end{cases} \tag{24}$$

Where $\lambda_p = \frac{\varepsilon-1}{\theta}$,$Kp = \left(\frac{\alpha}{1-\alpha}\right)\lambda_p$, $\lambda_w = \frac{\epsilon_w w}{\theta_w}$ and $k_w = \lambda_w\left[\sigma + \frac{\eta}{1-\alpha}\right]$

The proof of the proposition is straightforward (see equations 10, 18, 20, 21, and 22). There are two differences with the standard "3 equations" model. There is one Phillips curve for each source of nominal rigidity and the aggregation only alters the demand side of the model: the Dynamic IS equation and the change is proportional to the change in the wage gap. In the price NKPC, when real wage is above their natural level, firms reset price up because it increases their marginal cost. While in the wage NKPC, when real wage is above their natural level, the middleman resets nominal wage down to smooth the adjustment cost. The presence of Hand-to-Mouth(HtM) does not play any direct role in the first two equations. In the DIS the dynamics of the real wage gap matters for the aggregate response of the output gap to aggregate shock. In the absence of HtM (ie $\Psi_2 = \Psi_3 = 0$) this dynamics is no longer relevant. The reason is due to the fact that the HtM relies on their current income so the real wage. The change in their real wage appears to be very important for the aggregate response of an aggregate shock. In only sticky prices or only sticky wages, this real wage tends to be more volatile. Sticky wages and sticky prices together limit that volatility and thus the role of HtM in TANK.

Next proposition establishes our first main result. It states that RANK and TANK are equivalent in terms of aggregate fluctuations to aggregate shock.

Proposition 2 *Under sticky wages, RANK and TANK are equivalent. The TANK model can be summarized with the following system of 4 equations*

$$\begin{cases} \pi_t^w = \beta \mathbb{E}\pi_{t+1}^w + \lambda_w \left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right] \tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma}\left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma \Psi_a \mathbb{E}\Delta a_{t+1}\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \\ -\frac{\alpha}{1-\alpha}\Delta\tilde{y}_t = \pi_t^w - \pi_t^p & \text{Identity} \end{cases} \tag{25}$$

The system is independent of the share of HtM. Therefore, RANK is equivalent to TANK. The proof is as follows. First, use the previous proposition 1 and eliminate the Price NKPC equation (this equation does not hold under flexible price). Second, use the following relation $\tilde{w}_t = -\frac{\alpha}{1-\alpha}\tilde{y}_t$ and $\Psi_3 = \frac{\alpha}{1-\alpha}\Psi_2$. In monopolistic competition, the firm's profit is proportional to the price markup and output. Given that the profit rate (profit over output) depends only on the markup, the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market), is proportional to the price markup. If prices are flexible (and wages are sticky), the firm can always adjust its price for a constant price markup. Every firm faces the same nominal wage set by the wage union. So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK. In the case of sticky prices, we show in appendix B.1 that we are back to Debortoli and Galí (2018) case.

4 Findings: HANK vs TANK

In this section, I first present my calibration result with the method used to solve for the model. Second I outline some main findings with a discussion of Debortoli and Galí (2018) findings.

4.1 Calibration

In the calibration, for comparability reason, I stay as close as possible to Debortoli and Galí (2018). My model has two more parameters than Debortoli and Galí (2018), which are: elasticity of substitution across labor and the wage adjustment cost. I set the elasticity of substitution across labor to be $\epsilon_w = 10$ (the same as the elasticity of substitution among goods) and the wage adjustment cost to be $\theta_w = 150$ which is half of the value used in Hagedorn et al. (2019b)[7]. Time is set to be a quarter. I calibrate the discount factor β for the steady state risk-less real interest rate to be 3% per year. The borrowing limit is set to be $\Psi = 0.5$. This implies the share of constraints agents to be 21.7% (Wealth-based), 22.7% (Labor-based) and 26.8% (Uniform-based). The remaining parameters are the same as in (Debortoli and Galí (2018) section 3.5).

4.2 Numerical method

To solve for the TANK model, I use the system of equations in the proposition 1. This can be solved manually by guessing that each variable (control and state variables) is a linear function of exogenous state variables (monetary policy shock v_t, preference shock z_t, and technology shock a_t) and the endogenous state variable (real wage gap $\tilde{w}_{t-1}$). Let $Y_t = [\pi_t^p, \pi_t^w \ \tilde{y}_t \ \hat{r}_t]'$ be the vector of the control variables and $X_t = [\tilde{w}_{t-1} \ a_t \ z_t \ v_t]'$ be the vector of state variables (endogenous and exogenous states). $Y_t = g_x X_t$ and $X_{t+1} = h_x X_t$ where g_x is a matrix 4x4 and h_x is a matrix 4x4. With few seconds the matrix g_x and h_x can be computed using Schmitt-Grohé and Uribe (2004) toolbox.

The HANK model is solved in two steps. First, I solve for the stationary distribution in which every aggregate shock is equal to zero. Second, I solve for the dynamics in which I analyze the effect of each of the aggregate shock. I use Endogenous grid point method (see Carroll (2006)) to solve for the stationary distribution. The dynamics is solved by following Bayer and Luetticke (2018) and Bayer et al. (2019). The method is different from Debortoli and Galí (2018) [8]with almost the same result. This is an evidence of robustness of their finding. I discretize the idiosyncratic risk process using Tauchen (1986) with 11 grid points. I use 80 grid points for the net worth A_t and the minimum point is the corresponding borrowing limit. See appendix C for the details on the algorithm used to solve for the HANK model. Next section presents some of my main finding.

[7] This implies that wage is changing roughly every 2 quarters. This too low value is by purpose to show that our numerical result is not driven by a high degree of wage stickiness.

[8] They use the method outiline in Reiter (2009).

Table 1: Parameters

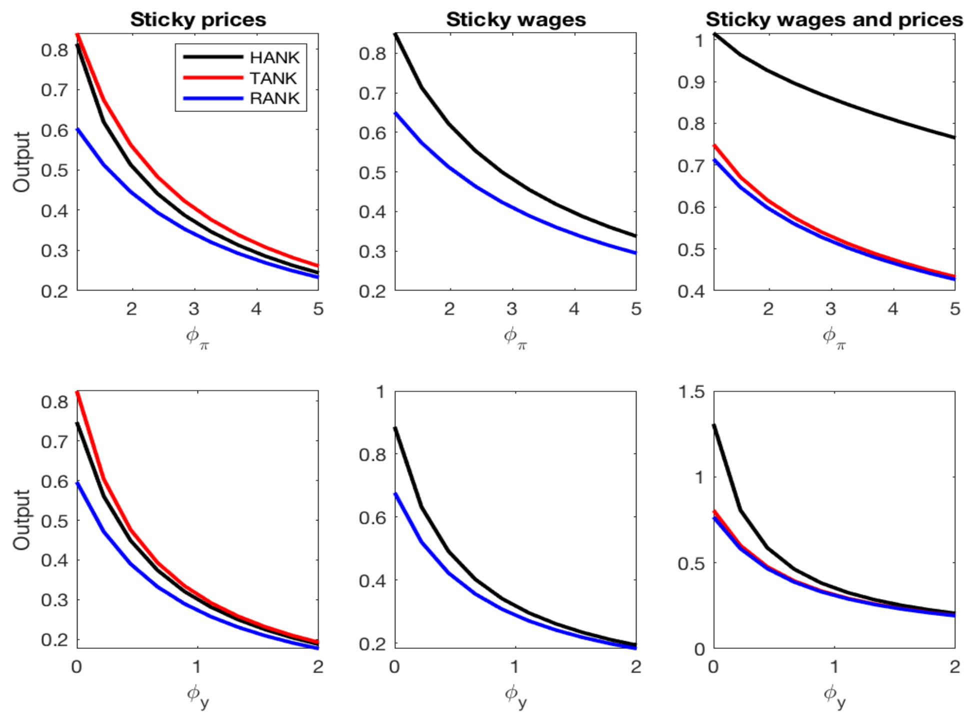
Parameter	Description	Target/source
$\beta = \begin{cases} 0.9778 \;\; W-rule \\ 0.9773 \;\; P-rule \\ 0.9799 \;\; U-rule \\ 0.9909 \;\; RANK/TANK \end{cases}$	Discount factor	avg real interest $\bar{r} = 3\%$
$\Psi = 0.5$	Borrowing limit	share of constr. 21.7% -26.8%
$\sigma = 1$	Risk aversion	Standard value
$\alpha = 0$	Curvature Prod. function	Standard value
$\eta = 1$	Frisch elasticity	Standard value
$\epsilon_w = 10$	Elasticity of substitution across labor	Standard value
$\epsilon_p = 10$	Elasticity of substitution among good	Profit share 10%
$\theta_p = 105.63$	Price adjustment cost	Debortoli and Galí (2018)
$\theta_w = 150$	Wage adjustment cost	slope of 0.06
$\rho_e = 0.9777$	Persistence of idiosyncratic shock	Debortoli and Galí (2018)
$\rho_v = \rho_z = 0.5$	Persistence of pref. and monetary policy shock	Debortoli and Galí (2018)
$\rho_a = 0.9$	Persistence of technology shock	Debortoli and Galí (2018)
$\phi_\pi = 1.5 \quad \phi_y = 0.5/4$	Interest rate coefficients	Debortoli and Galí (2018)
$\tau_a = 1; \;\; \tau_e = 0$	Wealth-based	Debortoli and Galí (2018)
$\tau_a = 0; \;\; \tau_e = 1$	Labor -based	Debortoli and Galí (2018)
$\tau_a = 0; \;\; \tau_e = 0$	Uniform -based	Debortoli and Galí (2018)

4.3 Findings

In this section I compare RANK, TANK and HANK model in terms of aggregate fluctuation following a monetary policy shock. Debortoli and Galí (2018) use three main outcome to compare RANK, TANK, and HANK model. First, they compare the path of the impulse response of aggregate variable (output, price inflation, real interest rate, etc.) following a demand shock and a supply shock. They conclude that the path in TANK closely follows the one in HANK. Second, they compare the path of the cumulative of the impulse response of the aggregate variable over 16 quarters after an aggregate shock for different values of the interest rate coefficients ϕ_π and ϕ_y. They conclude that the path in TANK track well the one in HANK. Third they simulate the model and compare some second moment (standard deviation, correlation) of the simulated time series of output (and output gap) and the heterogeneity factors of the models. They also conclude that in general, TANK approximates well HANK model[9].

The figure 1 makes it clear that in sticky prices, the TANK model approximates well the HANK model. In the

Figure 1: Change in environment, Monetary policy rule



Note: The figure compares the cumulative response over 16 quarters in sticky prices (first column), in sticky wages (second column) in sticky wages and prices(third column) for different values of ϕ_π (first row) and ϕ_y (second row) for RANK, TANK, and HANK.

presence of sticky wages and sticky prices TANK is far away from HANK. RANK instead approximates very well TANK model. It is another evidence that TANK is not so different from RANK in terms of aggregate fluctuations from monetary policy shock in the presence of both sticky wages and sticky prices.

My findings call for caution when comparing TANK to another model. In fact 10 years ago when TANK model becomes popular, Bilbiie (2008) emphasizes that under sticky prices for a plausible share of Hand to Mouth, the Taylor Principle is violated. Colciago (2011) introduces sticky wages in TANK find that the Taylor principle is

[9] See Debortoli and Galí (2018) section 5.2 for their general comment about TANK and RANK.

restored. Nowadays, HANK is becoming popular and Debortoli and Galí (2018) show that under sticky prices TANK approximates well HANK. In this paper, I show that this is no longer the case once we account for sticky wages. The reason why TANK becomes less powerful is that in the presence of both sticky wages and sticky prices, the real wage of the permanent Hand to Mouth Consumer fluctuates less. Also, TANK is equivalent to RANK under only sticky wages for the reasons I described.

4.4 Discussion

My paper highlights three main results. First, if prices are sticky (and wages are flexible), TANK approximates HANK. Second, if wages are sticky (and prices are flexible), TANK is equivalent to RANK. Then TANK cannot approximate HANK. Third, if prices and wages are sticky, TANK does not approximate HANK. It somewhat approximates RANK. It is crucial to know how the heterogeneity in HANK affects the aggregate consumption compare to the RANK to understand all those results.

The heterogeneity in HANK affects aggregate consumption in three dimensions: the Change in the consumption gap between unconstrained and constrained agents. This is the consumption inequality dynamics; the change in the consumption dispersion within the unconstrained agents; and the change in the share of constrained agents. Debortoli and Gali (2018) show that the first dimension is the most important difference between RANK and HANK. The key difference between RANK and HANK is the dynamics of consumption inequality (consumption gap between unconstrained and constrained agents). In TANK only the first dimension is (or maybe) present. By construction, the second and the third dimension are not present in TANK (or are always zero).

Given that the most important dimension of the heterogeneity in HANK is the consumption inequality dynamics and that the latter is present also in TANK, TANK can approximate HANK if it tracks well the consumption dynamics in HANK following an aggregate shock. Now, let explain what the sources of the consumption inequality in HANK and TANK are and discuss how the type of nominal rigidity affects the consumption inequality in TANK. In HANK, consumption inequality comes from labor income risk, firm's ownership, and debt choice. On the contrary, in TANK, the consumption inequality comes only from the firm's ownership, which can be understood as follows.

Economy 1: Suppose a continuum of identical agents of mass $1 - \alpha$ who consume and save into a risk-free asset. They have two sources of income: same labor income and same firm ownership income (firm's profit). Because every agent is identical, the choice of bond is zero at the equilibrium (the bond is not traded). So, every household will consume all his income at the equilibrium.Therefore, there is no consumption inequality.

Economy 2: Now, let's introduce in Economy 1 another type of agent of measure α who do not have access to the financial market (they cannot borrow/save). Both agents still have the same labor income and own the same share of firms. Then at the equilibrium, the consumption in economy 1 is the same as the one in economy two because again, the asset is not traded. Again, there is no consumption inequality, and the aggregate consumption will be the same. Despite the difference in the participation in the financial market, economy 1 and economy 2 are the same. This is because there is no trade in the bond and no difference in the budget constraint. So, let's call this result no-trade result.

Economy 3: Let modify economy 2. Let assume now that there is heterogeneity in the budget constraint so that only asset holders own the firms. Households who do not participate in the financial market receive only the same labor income as those who participate in the financial market. At the equilibrium, we still have no trade in the bond market. But now there Is a difference in the consumption of both types of agents. This will generate consumption inequality, a function of firm's profit.

In monopolistic competition, the firm's profit is given by the price markup. Given that the profit rate (profit over GDP) depends only on the markup, one can show that the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market) is proportional to the price markup.Note that our economy 3 is our TANK model, and Economy 1 is our RANK.

If prices are flexible (wages are sticky), the firm can always adjust its price for a constant price markup (every firm faces the same nominal wage set by the wage union). So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK (result 2). So why is HANK not equivalent to RANK when prices are flexible? The reason is simple. In HANK, the consumption inequality does not only come from the firm's ownership but also the labor idiosyncratic income risk and debt choice. If prices are sticky (wages are flexible), following an aggregate shock, the change in consumption inequality in TANK tracks well the one in HANK. So, TANK approximates HANK (result 1). Since prices are sticky, firms cannot adjust their prices to keep a constant markup, and then there is a movement in the consumption inequality. Numerically, this movement in the consumption inequality tracks well the one in HANK. Hence TANK can approximate well HANK. When both prices and wages are sticky, result 2 dominates result 1 so that TANK approximates RANK instead of HANK.

My results highlight the importance of the source of nominal rigidity. In a the standard New Keynesian literature, it is assumed that wages subject to some rigidities are set outside the firms. Wage unions have some labor market power which allow them to have some wage markup over the flexible wages. So every firm in the economy are homogeneous in wages. On the contrary, in the good market, firms have some monopolistic power which allows them to set prices subject to some rigidities. Firms are then ex-ante heterogenous in price setting.

Firms can only influence their profit via prices because they take wages are given. If firms were heterogenous in wages and can only set wages instead of prices, then TANK cannot be equivalent to RANK even if prices are flexible. Because what will determine Firm's profit now is how firms set wages.

5 Conclusion

This paper studies the implications of idiosyncratic income risk for the aggregate consumption responses to a monetary policy shock. In particular, the paper contrasts the properties of Two Agents New Keynesian (TANK) models with those of Heterogeneous Agents New Keynesian (HANK) models in a sticky wage environment in terms of aggregate fluctuations.

First, I show that under sticky wages (and flexible prices) the TANK model is equivalent to the RANK model. This equivalence means that the equations summarize the state of the economy in a TANK model do not depend on the share of "hand-to-mouth" consumers. In this environment, the consumption inequality is constant over time so not relevant following an aggregate shock. It follows that the TANK model cannot approximate HANK models. Second, under sticky prices (and flexible wages), the TANK model can approximates well HANK model as shown by Debortoli and Galí (2018). But in a sticky prices and sticky wages environment, the TANK model can no longer approximate HANK models in terms of aggregate fluctuations.

My findings call for caution when comparing TANK to other models in a nominal rigidity environment. In fact, how TANK model performs greatly depends on the type of nominal rigidities. With inequality, not only the type of nominal rigidity matters but also the source of nominal rigidity because redistribution matters. Wage markup is uniformly redistributed to every household in my work because wages are set outside the firms. On the contrary, price markup is not redistributed uniformly. In general, only asset holders gain from a price markup since they earn the firms.

In addition, central banks worldwide that aims to integrate income inequality in their quantitative framework, should investigate the source of nominal rigidity in their economy.

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A Firm problem

A.1 Profit Maximisation: Employment Agency

$$\max_{N_{it}} W_t N_t \quad - \quad \int_0^1 W_{it} N_{it} e_{it} \tag{26}$$

$$s.t \quad N_t \quad = \quad \left[\int_0^1 e_{it} \left(N_{it} \right)^{1-\frac{1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{27}$$

CPO:

$[Nit] : W_t \frac{\partial N_t}{\partial N_{it}} - W_{it} e_{it} = 0$ where $\frac{\partial N_t}{\partial N_{it}} = e_{it} N_{it}^{-\frac{1}{\epsilon_w}} N_t^{\frac{1}{\epsilon_w}}$. Then it follows the the demand for the i-th consumer's labor in the main text 4 .

A.2 Price decision of Intermediate good produce

$$\max_{p_{j,t+s}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s Q_{t+s/t} \left\{ \left(\frac{p_{j,t+s}}{P_{t+s}} - m_{j,t+s} \right) y_{j,t+s} - \frac{\theta}{2} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \pi \right)^2 Y_{t+s} \right\} \tag{28}$$

$$st \qquad y_{j,t+s} = \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \tag{29}$$

FOC

$$\begin{aligned} & \mathbb{E}_t \beta^s Q_{t+s/t} \left[\frac{1}{P_{t+s}} y_{j,t+s} - \frac{\varepsilon}{P_{t+s}} \left(\frac{p_{j,t+s}}{P_{t+s}} - m_{j,t+s} \right) y_{j,t+s} \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon-1} Y_{t+s} - \theta \frac{1}{p_{j,t+s-1}} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \pi \right) Y_{t+s} \right] \\ + \quad & \mathbb{E}_t \beta^{s+1} Q_{t+s+1/t} \left[\theta \frac{p_{j,t+s+1}}{p_{j,t+s}^2} \left(\frac{p_{j,t+s+1}}{p_{j,t+s}} - \pi \right) Y_{t+s+1} \right] = 0 \end{aligned}$$

Using symetric price $p_{j,t} = P_t$ we have $y_{j,t} = Y_t$. Using the definition for the inflation $\Pi_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}$ and reaarring the FOC we get:

$$\mathbb{E}_t \left[[1 - \varepsilon m_{t,s}) - \theta \left(\Pi_{t+s} - \Pi \right) \Pi_{t+s}] - \beta\theta \Lambda_{t,t+s+1} \left[\Pi_{t+s+1} \left(\Pi_{t+s+1} - \Pi \right) \frac{Y_{t+s+1}}{Y_{t+s}} \right] \right] \tag{30}$$

where $\Lambda_{t,t+s+1} = \frac{Q_{t+s/t}}{Q_{t+s+1/t}}$. The above equation is true for every s. For $s = 0$ and the steady inflation $\Pi = 1$ we have :

$$\Pi_t(\Pi_t - 1) = \frac{1}{\theta} - \frac{\varepsilon}{\theta}(1 - m_t) + \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right] \tag{31}$$

Log linearize the above around the steady state we get:

First taylor approximation of the LHS

$$\begin{aligned} \Pi_t(\Pi_t - 1) &\simeq \Pi(\Pi - 1) + (2\Pi - 1)(\Pi_t - \Pi) \\ &= 0 + (\Pi_t - 1) \\ &= \pi_t \end{aligned}$$

First taylor approximation of the RHS

$$\begin{aligned} &\simeq 0 + \frac{\varepsilon}{\theta}(m_t - m) + \beta \mathbb{E}\left[\Lambda \frac{Y}{Y} (2\Pi - 1)(\Pi_{t+1} - \Pi)) \right] \\ &= \frac{\varepsilon}{\theta}(m_t - m) + \beta \mathbb{E}(\Pi_{t+1} - \Pi) \\ &= \frac{\varepsilon}{\theta}(m_t - m) + \beta \mathbb{E}(\Pi_{t+1} - 1) \\ &= \frac{\varepsilon m}{\theta} \hat{m}_t + \beta \mathbb{E} \pi_{t+1} \end{aligned}$$

Equating both side we get:

$$\pi_t = \beta \mathbb{E} \pi_{t+1} + \frac{\varepsilon m}{\theta} \hat{m}_t \tag{32}$$

Note that $m = \frac{\varepsilon - 1}{\varepsilon}$. $\hat{m}_t$ is the log deviation of the marginal cost from his steady state and $-\hat{m}_t$ is the log deviation of firm markup from his steady state. Equation 9 says that if firm markup is below their natural level then price will increase (vis-versa).

B Household problem

B.1 TANK: sticky prices

$$\begin{cases} \pi_t^p = \beta \mathbb{E}\pi_{t+1}^p + \lambda_p \tilde{w}_t + k_p \tilde{y}_t & \text{NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1+\Psi_3)} \left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma \Psi_a \mathbb{E}\Delta a_{t+1}\right] + \frac{\Psi_2}{1+\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \end{cases} \tag{33}$$

Using the following relation $\tilde{w}_t = \left[\sigma + \frac{\eta}{1-\alpha}\right] \tilde{y}_t$ and $\Psi_3 = \frac{\alpha}{1-\alpha}\Psi_2$, it is straightforward to end up with the following system of equations:

$$\begin{cases} \pi_t^p = \beta \mathbb{E}\pi_{t+1}^p + \lambda_p \left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right] \tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma} \frac{1}{1+(\sigma+\frac{\eta+\alpha}{1-\alpha})\Psi_2} \left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma \Psi_a \mathbb{E}\Delta a_{t+1}\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \end{cases} \tag{34}$$

As opposed to the sticky wage framework, the system is not independent of the proportion of hand to mouth meaning that the RANK is not equivalent to TANK under sticky prices. From the DIS equation, if $1 + (\sigma + \frac{\eta+\alpha}{1-\alpha})\Psi_2 < 0$, a positive shock on nominal interest rate lead to boom: what Bilbiee (2008) refers to the Inverted Aggregate Demand Logic (IADL) region.

B.2 Proof of Lemma 2

The goal of this section is to linearize **??** to find an anlytical expression $\hat{\gamma}_t$. [10]

$C_t^U - C_t^K = D_t \left(\frac{1-(1-\tau)\delta}{1-\lambda}\right)$ and $(1-\lambda)C_t^U = (1-\lambda)w_t N_t + (1 - \delta\lambda(1-\tau))\, D_t - (1-\lambda)AC_t^w$

$$\gamma_t = \frac{D_t\,(1-(1-\tau)\delta)}{(1-\lambda)w_t N_t + (1-\delta\lambda(1-\tau))\,D_t - (1-\lambda)AC_t^w} \tag{35}$$

The profit $D_t = Y_t - w_t N_t - AC_t$. Since $m_t = \frac{w_t}{MPN}$ we have $w_t N_t = (1-\alpha)\, m_t Y_t$ .So $D_t = \frac{1}{(1-\alpha)m_t} w_t N_t \left(1 - \tilde{AC}_t\right) - w_t N_t$ where $\tilde{AC}_t = \frac{\theta}{2}\left(\Pi_t^p - \Pi^p\right)^2$. γ_t can be rewritten as :

$$\gamma_t = \frac{\left[\frac{1}{(1-\alpha)m_t}\left(1 - \tilde{AC}_t\right) - 1\right](1-(1-\tau)\delta)}{1 - \lambda + (1-\delta\lambda(1-\tau))\left[\frac{1}{(1-\alpha)m_t}\left(1-\tilde{AC}_t\right) - 1\right] - (1-\lambda)\frac{AC_t^w}{W_t N_t}} \tag{36}$$

[10] See Debortoli and Galí (2018) Section 5

The steady state value of γ_t is given by: $\gamma = \frac{[1-(1-\alpha)m](1-(1-\tau)\delta)}{(1-\lambda)(1-\alpha)m+(1-\delta\lambda(1-\tau))[1-(1-\alpha)m]}$. Let's γ_m be the first partial derivative of γ_t evaluated at the steady state.

$$\gamma_m = -\frac{(1-\alpha)(1-\lambda)(1-\delta\lambda(1-\tau))}{[(1-\lambda)(1-\alpha)m + (1-\delta\lambda(1-\tau))[1-(1-\alpha)m]]^2} \tag{37}$$

So linearizing γ_t, we get:

$$\hat{\gamma}_t = \Psi_1 \hat{\mu}_t^p, \tag{38}$$

where $\Psi_1 = -\gamma_m m$; $\Psi_1 > 0$ and $\hat{\gamma}_t = \gamma_t - \gamma$

C Numerical method

C.1 Stationnary distribution

Prelimilary:

Construct a $ne = 11$ grid point for e and $na = 80$grid point for the asset A. I use a log-space (not a linear) grid point for the asset.

Define $C_{ij} + A_j = X_{ij}$. Where X_{ij} is a cash on hand for an household with idiosyncratic risk e_i and an asset A_j. X_{ij} is composed of labor income, bond income, equity income and income and transfer income minus the wage adjustment cost.

$X_{ij} = w.Ne_i + (1+r)A_j + [-(1+r)Q + Q + (1-\delta)D]\frac{A_j^+}{A^+} + T_{ij} - AC_i^w$ where $T_{ij} = \left[1 + \tau^a\left(\frac{A_j^+}{A^+} - 1\right) + \tau^e(e_i - 1)\right]\delta D$

$A_j^+ = \max[0, A_j]$. A^+ is the total asset hold by positive asset holders

Compute some aggregate steady state variables which do not require a distribution given my model.

$$mc = \frac{\epsilon_p - 1}{\epsilon_p} \tag{39}$$

$$\mu_w = \frac{\epsilon_w}{\epsilon_w - 1} \tag{40}$$

$$N = [(1-\alpha)mc.\mu_w]^{1/(\sigma(1-\alpha)+\alpha+\eta))} \tag{41}$$

$$w = (1-\alpha)mc.N^{-\alpha} \tag{42}$$

$$Y = N^{1-\alpha} \tag{43}$$

$$D = Y - w.N \tag{44}$$

$$Q = \beta[Q + (1-\delta)D] \tag{45}$$

1. Guess A^+

(a) Guess β

i. Guess the consumption $C_{ij} = w.Ne_i + [-rQ + (1-\delta)D]\frac{A_j^+}{A^+} + T_{ij}$ for every i and j. Let's denote it C_{guess}

A. Update 1(a)i using the Euler Equation (with equality) Let's denote it C_{new}

B. Compute the policies function A_{ij}^*

C. Identify binding constraints

D. Interpolate the policies function A_{ij}^* and C_{new} on A grid . Denote the policy function A_{star}

E. Update 1(a)iD by taking into account the binding constraints . For binding constraints we have $C_{ij} = X_{ij} - min(A_j)$. Let denote this C_{star}

F. if $\max(abs(C_{guess} - C_{star}))$ close to zero enough . stop if not update $C_{guess} = C_{star}$ and go back to step 1(a)iA

ii. Use A_{star} to compute the stationary distribution μ

(b) Check the asset market $\sum_i \sum_j A_{star}^{ij}\mu_{ij} = Q$ If the asset market verified, stop. If not go back to step 1a

2. Compute $A_{new}^+ = \sum_i \sum_j A_j\mu_{ij}(A_j > 0)$. If A_{new}^+ enough close to A^+, stop if not go back to step 1

C.2 Aggregate fluctuations

I closely follow Bayer et al. (2019) to solve for the aggregate fluctuation. The HANK model can be summarized in a system of equations of the form

$$\mathbb{E}\left[X_t, X_{t+1}, Y_t, Y_{t+1}\right] = 0 \tag{46}$$

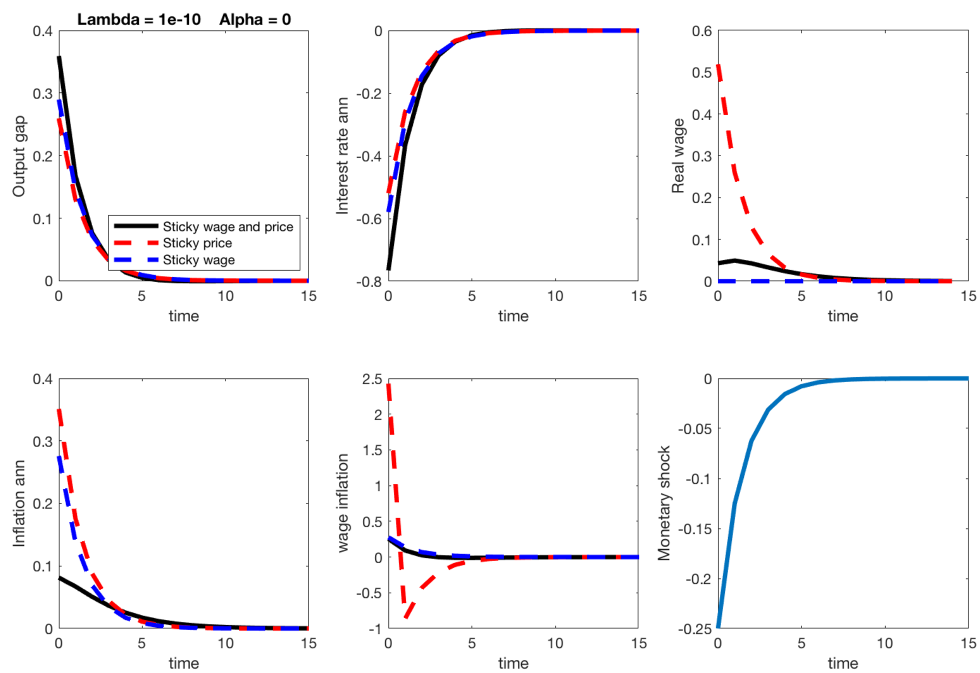
where X_t is a set of state variables and Y_t is a set of control variables. This can be solved using Schmitt-Grohé and Uribe (2004) toolbox. Bayer et al. (2019) propose a matlab file (a variant of Schmitt-Grohé and Uribe (2004) algorithm to solve for the system). Note that the number of state variables here is 80x11-1 + 3. 80x11-1 is the number of state variables from the joint distribution of asset and labor income risk. 3 is the number of aggregate states variable $(R_t, w_{t-1}, v_t/z_t/a_t)$. The number of control variables is 80x11+7. 80x11 for each level of consumption and 7 for the number of aggregate control variables ($y_t, \pi_t^p, \pi_t^w, Q_t, D_t, N_t, A_t^+$). So 46 is a system of 2x80x11-1+3+7 = 1769 equations. Bayer et al. (2019) proposes a method to reduce the dimension of the state and the control variables. For details on the reduction of the dimensionality please see (Bayer and Luetticke (2018) and Bayer et al. (2019)

1. I solve my system without applying of the reduction of the dimension proposed by (Bayer and Luetticke (2018) and Bayer et al. (2019) . That is the full system of 1769 equations.
2. I solve my system by reducing just the dimension of the state variable. This gives a system of (80+11)-2 + 3 +80x11+7= 979 equations
3. I solve the system by reducing both the state space and the control space. This gives a system of 164 equations.

I find (almost) no difference in the impulse response of aggregate control variables.

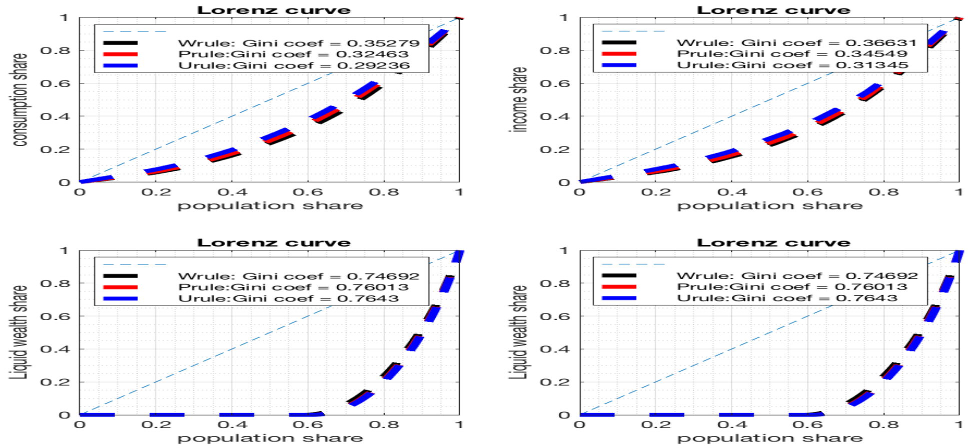
D Monetary policy (MP) shock

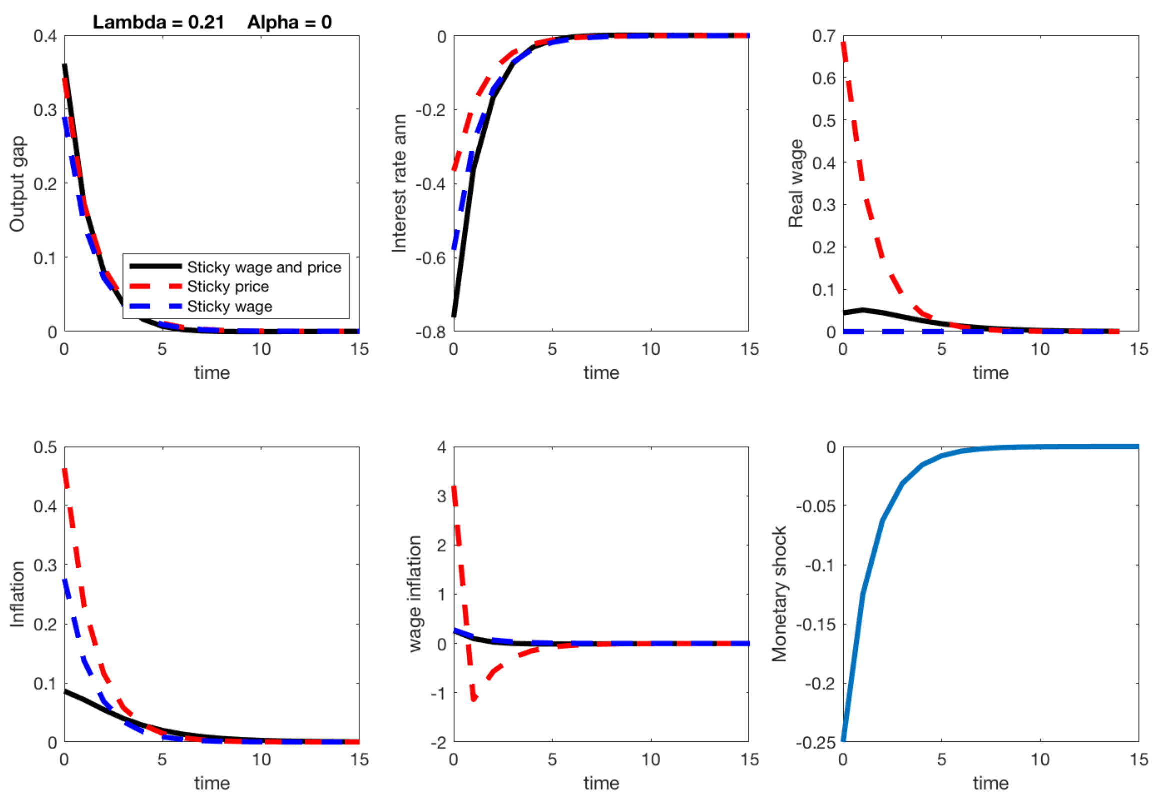
D.1 MP shock: RANK



D.2 Solution for stationary distribution

D.3 MP shock: TANK


Lorenz curve
Wrule: Gini coef = 0.35279
Prule:Gini coef = 0.32463
Urule:Gini coef = 0.29236
consumption share
population share
Lorenz curve
Wrule: Gini coef = 0.36631
Prule:Gini coef = 0.34549
Urule:Gini coef = 0.31345
income share
population share
Lorenz curve
Wrule: Gini coef = 0.74692
Prule:Gini coef = 0.76013
Urule:Gini coef = 0.7643
Liquid wealth share
population share
Lorenz curve
Wrule: Gini coef = 0.74692
Prule:Gini coef = 0.76013
Urule:Gini coef = 0.7643
Liquid wealth share
population share


Lambda = 0.21 Alpha = 0
Output gap
Sticky wage and price
Sticky price
Sticky wage
time
Interest rate ann
time
Real wage
time
Inflation
time
wage inflation
time
Monetary shock
time

D.4 Aggregate fluctuations: IRF of MP shock